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EARLY IMPRESSIONS OF THE TEACHING OF MATHEMATICS IN THE SECONDARY SCHOOLS OF PARIS.

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The year 1909-10 being his year out of residence was spent by the writer in visiting elementary and secondary schools in Europe to acquaint himself directly with the actual practice of mathematical teaching. Four months were given to the public schools of Paris; two months to those of Berlin and Charlottenburg, and two months to London schools. Early impressions are likely to be of doubtful value, but although this title has been chosen as a caption under which some remarks are to be made to teachers the visiting reported upon ended last May, and the succeeding months have exercised some seasoning influence upon the impressions. They cannot now with good grace be regarded as first impressions. Perhaps it will be as well for the reader to think of what is said merely as impressions of a teacher about foreign teaching of his special subject, and allow them to count for as much or as little as such impressions may be worth.

Foreign educational practice may be studied in either of two distinct ways, dependent upon the purpose of the studying. The entire field of public education may be surveyed and examined in such detail as may be necessary to give a fairly complete picture of the general *status* at a particular epoch, or the field of observation and examination may be narrowed to a more intensive study of class-room practice in a certain subject, and in certain institutions chosen with regard to typical educational purpose and character with a view to getting at the actuating ideals and the energizing spirit of plans and programs as these reveal themselves in practice. Both on account of limited time and of specific purpose had in view, the writer chose the second type of study. If we may regard the educational duty of the representative of a special subject in the secondary school to be merely to view the whole field of school work, and to exploit the entire

range of educational materials from a particular point of view, the second type of study need not result in a narrowly specialized outcome.

The institutions selected for examination were the public elementary and secondary schools of the capital cities of France, Germany, and Great Britain. The subject to which attention was rather narrowly confined was the teaching of mathematics in these institutions. The writer held it most consonant with his purpose to study *good* class work, perhaps the very best that the educational systems of these countries are producing. He therefore requested the authorities to give him leave to visit the institutions best exemplifying the varied phases of mathematical education that prepared for the various higher lines of study in the two classes of institutions mentioned. He was told more than once in Paris that the institutions chosen for him were the most nearly typical of the best in mathematical education that could be found. He found also that the official heads of the institutions visited commonly regarded it as a compliment to have been put on his list by the vice-rector.

There are many important educational functions of mathematical study in the new and special types of schools that are forming in all large civic centers in Europe along vocational lines. These uses seem to me, however, to be of a specialized and subsidiary nature, and of distinctly less significance than the main educational function of mathematics of the undifferentiated and unspecialized programs to which nine out of ten must subject themselves.

The educational service of mathematics in these new vocational and trade schools might well constitute a study in itself, and would amply repay a trip abroad. But the writer's interest was in the "middle-of-the-road" duty of public-school mathematics, and he found that his time was all too short to discharge even that duty fittingly.

Just here it may be said that there has been too much dilettante flitting about over Europe by American teachers, oftentimes merely college students, "doing" cities educationally at the rate of two or three large ones a week, and after two or three months coming home with the sum and substance of educational Europe *in one grip*, and then posing before us as authorities on European education. This is not said to impress the reader with the greater depth and closer truthfulness to fact of what the writer is to say later, but merely as a matter of information to whom it may con-

cern that he may be less likely to be fooled by this species of American "educational aviation" in Europe. Beware of the man who can write a fat volume on *"Educational Germany"* from a summer's visit, mostly for pleasure, and who can sum up his findings of German mathematical teaching with: "American teachers have nothing to learn from Germany on the teaching of mathematics." Such conclusions are born of self-conceit, or self-deception, or prejudice, and are unworthy of reliance. I have to say that German teachers and teaching of mathematics are far and away better than anything else in Europe, and better than anything that can be easily found in America. If American teachers have nothing to learn from German mathematical teaching, the fault is not with the German teaching. But German teaching is not the theme of this paper. The same is moreover true of French teaching.

Again, it seems to the writer that it is true of both Europe and America that if the energy spent on developing special schools for a comparatively few pupils were spent on the less spectacular but more useful work of making the general school work to fit better the demands of twentieth century life, everybody would be the gainer. In many ways it is a fact that instead of school programs coming closer to the needs of daily life, each year finds the discrepancy wider than the last. In Europe as in America school officials seem too much concerned about maintaining the existing *status,* either because they understand it and have learned how to function in it, or because some official may have his administration discredited if something is improved. The writer has long been of the opinion that administrative officers in the public schools are often obstructions to progress in improving mathematical teaching. Often they seem also to be the foes of even meritorious improvement in systems at large if it unsettles what they look upon as settled. The administrator is too frequently not an organizer of work but a *settler* of difficulties.

In the Schools of Paris.

The first thing to impress the writer in Paris was the readiness of the vice-rector of the academy to furnish every means to becoming acquainted with the actual class work of mathematics teaching in the schools both of Paris and elsewhere, if authorization elsewhere were wished. A letter of request accompanied by a note of introduction from the office of the United States Commissioner of Education, left with the secretary on Friday, brought

on the following Monday full authorization to visit mathematical classes in a list of schools chosen for me by the vice-rector as best exemplifying what I had requested to see. For this courtesy as well as for the courtesy shown me by the various *proviseurs* and *directeurs,* who on presenting this authorization gave me every means consistent with the rules of the schools to acquaint myself with the work, very cordial acknowledgments are hereby expressed. A rule of the administration that a teacher must be notified beforehand of the visitor's intention to visit him, prevented the writer from seeing any work unheralded. He does not believe, however, that what was seen was in many cases "cooked up" for "showcase" use. Naturally, he cannot be certain on this point.

The writer was surprised to find the class-room equipment for mathematical teaching so meager, but was gratified to see the full use that was made of such equipment as was available. Blackboards were small and usually one sufficed for a class room. There were blackboard rulers, compasses, protractors, and often a supply of mathematical models. These were used continually in the class work by both teachers and pupils. Their chief office was not merely to gather museum dust.

Nothing impressed the writer more strongly than the general absence of text-books from the classes. The only books in sight were the pupils' notebooks, in which the pupils of the older classes were as industrious to record what the professor was saying as are the older students of American universities. The writer was told that the main reliance of the pupil must be upon the notes taken in class; but that in his home or, if he be an *interne,* in his room he has a text, indicated by the professor, and that in this text he may look up matters to perfect his notes and his understanding of what has been gone over in class. The note-taking begins with the classes of ten year olds in arithmetical work, although in the earlier classes the teacher talks very slowly, repeating important truths, often two or three times. Often, perhaps generaliy, the teacher calls on one or more pupils to read what they have written, that he may correct any errors in the written records. Great pains are taken with beginners at this work to teach them how to take notes. The teacher often stops and tells pupils what particular thing it is important that they shall get into their notes correctly. The general plan with early classes, after the two or three years of rapid mental calculation,

is then a sort of teaching by dictation, while with later classes it is lecturing, pure and simple.

The method generally followed in presenting new subjects is that of direct exposition on the teaching side, and of remembering on the learning side. Teachers do not generally defend this plan on pedagogic grounds, but say that the distended condition of programs makes it a practical necessity as being the quickest way of covering the required topics for which the allotted time is too short at best. There is much talk that is friendly to the heuristic plan as practiced in Germany, but there is as yet comparatively little actual use of this plan. A few teachers called their plan a conversational method. In classes taught by these teachers there is rather a freer and more informal talking about points of particular difficulty than is customary. But even here the teacher gravitated quickly and almost invariably into the lecture and exposition procedure. As with us the lecture plan with boys and girls is roundly condemned and extensively used. The older the teacher the more does he resort to lecturing. This again is true with us.

A point that impressed me early and persistently was the very serious attitude assumed by pupils toward mathematical study. Boys and girls seem to feel they must learn their mathematics thoroughly. They work at it like little Trojans. They are visibly mortified by mistakes and failures, and do their best both to avoid them and to atone for them, once they are made. There was nowhere manifest a disposition to lay the blame for the pupil's academic shortcomings on the teacher, or the school. The pupil readily admitted the delinquency to be his own, and a disgrace both to himself and to his teacher. This spirit, in my opinion, arises from the attitude of teachers and parents toward the importance of the school tasks. Teachers are tremendous workers. They throw themselves heart and soul into their teaching. Either through officials specially appointed for the duty or through their own efforts, they charge themselves with an all-around concern for the intellectual welfare of their pupils. They do concern themselves about his progress, and make this concern felt by the pupils themselves.

Full professors are liable to ten hours' teaching per week in the most advanced classes, and others must teach from twelve to fifteen hours per week. Acting professors under fifty are liable for an hour more than the older professors, and anyone may be called upon, if he is needed, for two supplementary hours,

for which he is paid extra. In Paris teachers of mathematics are nearly always called upon for the supplementary hours, and considering the high cost of living in this great city, the teachers are usually very ready to render the additional service. The bulk of the work of personal surveillance of pupils is done by tutors and younger officials appointed especially for this service. This relieves the regular professor's daily program considerably, and on the whole allows him more time for his own purposes than is allowed the American secondary teacher. At the same time, it brings about much more personal supervision of pupils. Indeed, it seemed to me that French boys and girls are oversupervised.

Secondary teachers have all had the same academic and professional training as the university professors. They have passed the same examinations and are capable of entering upon university teaching should the opportunity arise. The opportunity, moreover, does often arise. Many of the professors of mathematics of the *Sorbonne* (the University of Paris) have served an apprenticeship in the secondary schools of the country before coming to the university. They have been chosen to their present posts because of especially meritorious services in the secondary field. The preparation of secondary and elementary school programs for the whole country by the university authorities seems less of an absurdity in the light of these facts. In general the faculties of universities are better informed on secondary work in France than in America.

It is difficult to form any useful idea of the average program allotment of time to mathematical study in our public schools. Some schools minimize the time for mathematics. Some assign the subject no program time in the first grade, some assign none in either the first or second grade, and a few begin the subject in the fifth or sixth grade. Recitation periods also differ greatly amongst us. Still an attempt to compare the time given in our schools to mathematical study with that assigned it by the French programs seems worth while.

Estimating as closely as one well can a fair average time allotment to mathematics in our public schools, we may say that out of a six-hour school day mathematics gets—

in the first and second grades, 20 minutes, or 5.5% of the program time,

in the third to the sixth grades, 30 minutes, or 8.3% of the time,

in the seventh and eighth grades, 45 minutes, or 12.5% of it,

in the first and second high school years, 55 minutes, or 18.3% of it, and

in the third high school year 28 minutes, or 9.1%.

These results are summarized in the accompanying table of

PER CENTS OF SCHOOL PROGRAM TIME FOR MATHEMATICS IN THE UNITED STATES AND IN FRANCE.

AGE.	IN U. S.	IN FRANCE.			
6	5.5%	14.3%			
7	5.5%	14.3%			
8	8.3%	22.8%			
9	8.3%	22.8%			
		A	B		
10	8.3%	9.6%	24%		
11	8.3%	9.6%	22%		
12	12.5%	14.4%	22%		
13	12.5%	14.4%	22%		
		A	B	C	D
14	18.3%	8.5%	12.5%	23%	48%
15	18.3%	4.6%	4%	24%	48%
16	9.1%	9%	9%	33%	32%

If the pupil is destined to any career in which mathematics plays an important role, the column of per cents farthest to the right indicates the per cents of his program time to be devoted to mathematical study. The reader may make his own comparisons and draw his own morals.

The writer's experience in visiting does not lead him to feel that French teaching has very much help for American teachers *from the viewpoint of methodology*. But from the point of view of academic attainment the results accomplished in their pupils are indeed impressive. It is stating the case with moderation to say that at the end of the secondary course, which the gifted boy may complete at sixteen years of age, the French boy is mathematically about as well off as our university sophomores at the end of the sophomore year, save as to calculus, and that by the time certain reforms now well along are completed even this reservation will not need to be made. In higher algebra, in the theories of equations and of functions, he even now has the mathematical equivalents of what he lacks in the calculus. These results are, as the writer looks at it, accomplished in spite of a very certain and great lack of efficient methods. They are the fruits of serious purpose and hard work on specifically indicated subject matter. To know definitely what one has to accomplish and to drive directly for it certainly leads to a commendable measure of results. With this must be included also

the advantage of teachers that are academically highly trained and efficient according to academic standards. Still one wonders where to see exemplified that excellent doctrine that originated in France that it is the *way of the thing* rather than *the thing itseif* that really educates.

It should at least be mentioned here that the immense simplification of arithmetic teaching due to the use of the metric system of standards was a conclusive argument for its adoption. These weights and measures adapt themselves so perfectly to our number notation and operations, that the out of school knowledge the pupil has of the standards is capitalized at its full value in the arithmetic. A skeptic as to the desirabilty of its use amongst us would be transformed to a believer by visiting a few classes in French school arithmetic.

It is not an easy task to unravel from the tangle of practice the educational ideals out of which the practice proceeds. One hears much of the same sort of talk about the educational value and significance of mathematical study in France as in America. As in America so also in France, practice and profession are not the same thing. In the writer's judgment the doctrine of formal discipline is in better standing among French teachers than among American teachers. Discipline, culture, training, and technical skill, with some regard for utility as a subsidiary end, are the ideals back of most of the teaching practice. And all these are to be sought in the matter rather than in the manner of teaching it.

One very good teacher says: "For the very great majority of our professors the chief end of mathematical teaching is that of forming the mind, the development in the pupil of the logical faculty. At the same time among teachers of the later classes, especially those containing students specializing for mathematics, there will be found mingled with this aim that of creating a taste for mathematics. This latter point of view is explicitly accepted for the classes called *Mathématiques speciales* (special students of mathematics)." The idea of the value of mathematical education as a preparation for lifework has but little acceptance in France. A few teachers, who are also authors of texts in school use, told the writer that the need for practical problems for use in teaching mathematics was coming to be felt very keenly, and one or two mentioned the work of SCHOOL SCIENCE AND MATHEMATICS along this line with great enthusiasm.

The reforms now on the point of making are:

For analysis, the introduction of the function idea as an organizing center, the introduction of both differential and integral calculus in the secondary schools, and the early and continuous use of the graph, and of graphical procedures.

For geometry, the simultaneous teaching of plane and solid geometry and trigonometry, the extensive use of motion in geometry, and the close correlation of geometry and algebra.

Some of these reforms, notably those pertaining to geometry, are already far along; while those for algebra are just now being made. The highly centralized plan of the French educational system makes the work of introducing reforms practically that of convincing the central office that they should be made. When the decree issues the reform is forthwith a fact. The high proficiency of teachers takes care of practical difficulties.

ON "FORMULAE FOR RATIONAL RIGHT TRIANGLES."

By Artemas Martin, LL.D.,
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"Lest we forget."—*Kipling.*

Lest those readers of School Science and Mathematics who have not made a special study of right-angled triangles might think the formulas given in the table on p. 683 of the November No., Vol. X., 1910, are distinct and independent, perhaps it would be well to show that all the others can be deduced from or reduced to No. 5 which, to avoid confusion of letters, let us change to $2pq$, $p^2 - q^2$, $p^2 + q^2$.

Now put $p = a + 1$, $q = a$, and we have $2pq = 2a\ (a + 1)$, $p^2 - q^2 = 2a + 1$, $p^2 + q^2 = 2a^2 + 2a + 1$, which is the same as No. 1.

Take $p = 2a$, $q = 1$; then $2pq = 4a$, $p^2 - q^2 = 4a^2 - 1$, $p^2 + q^2 = 4a^2 + 1$, the same as No. 2.

Take $p = \sqrt{(\frac{1}{2}a)}$, $q = \sqrt{(\frac{1}{2}b)}$; then $2pq = \sqrt{(ab)}$, $p^2 - q^2 = \frac{1}{2}(a - b)$, $p^2 + q^2 = \frac{1}{2}(a + b$, which is No. 3.

No. 4 may be divided through by a and then written

$$\frac{2b}{b^2+1},\ \frac{b^2-1}{b^2+1},\ 1 \text{ or } \frac{b^2+1}{b^2+1};$$

or, dropping the common denominator b^2+1,

$$2b,\ b^2-1,\ b^2+1,$$

which will be produced by No. 5 if we take $p = b, q = 1$.

Take $p = \sqrt{\left(\frac{a}{2b}\right)}$, $q = \sqrt{\left(\frac{b}{2}\right)}$; then $2pq = \sqrt{a}$,

$p^2 - q^2 = \frac{1}{2}\left(\frac{a}{b} - b\right)$, $p^2 + q^2 = \frac{1}{2}\left(\frac{a}{b} + b\right)$, which is the same as No. 6.

Dividing No. 7 through by a it may be written

$$1 \text{ or } \frac{b^2-1}{b^2-1}, \frac{2b}{b^2-1}, \frac{b^2+1}{b^2-1},$$

which, by dropping the common denominator b^2-1, may be written

$$2b, b^2 - 1, b^2 + 1,$$

the same as obtained from No. 4 and therefore is reducible to No. 5.

No. 8 can be written

$$\frac{ab}{b}, \frac{1}{2}\left(\frac{a^2}{b} - b\right), \frac{1}{2}\left(\frac{a^2}{b} + b\right); \text{ or } \frac{2ab}{b}, \frac{a^2-b^2}{b}, \frac{a^2+b^2}{b}.$$

Dropping the common denominator b we have No. 5.

No. 9 is simply a multiple of No. 5.

No. 5 will give all possible right-angled triangles whose sides are whole numbers prime to each other if a and b are prime to each other, one odd and the other even; and No. 9 will give all the multiples of the prime triangles.

Nos. 1, 2, 4, and 7 are only special cases of No. 5 and will give only certain classes of triangles.

In *all* the triangles given by No. 1, the hypotenuse and one leg differ by unity, or are consecutive numbers.

In *all* the triangles given by Nos. 2, 4, and 7, the hypotenuse and one leg differ by 2.

In No. 4, column "B," and in No. 7, column "$\sqrt{(A^2 + B^2)}$," the sign of division does not "show up" between the numerator and denominator.

In a paper on Rational Right-Angled Triangles published in the *Mathematical Magazine,* Vol. II., No. 12, I have given on pp. 301-308 an extensive table of prime rational right-angled triangles; on p. 322, a table of rational right-angled triangles whose legs are consecutive whole numbers; on p. 323, a table of rational right-angled triangles whose hypotenuse and one leg are consecutive whole numbers; and on p. 324, a table of rational right-angled triangles whose hypotenuse exceeds one leg by 2.

SHOULD CONCRETE MULTIPLIERS AND DIVISORS BE ALLOWED?

In a recent article on the above subject,[1] Mr. P. G. Agnew, of the National Bureau of Standards, raises a question which deserves careful consideration by all secondary school teachers of physics and mathematics, as it relates to the methods to be employed in the lower grades, but it is written primarily from the standpoint of the high school teacher.

Such forms as

$$\$90 \div 5 \text{ hogs} = \$18 \text{ per hog,}$$
$$75 \text{ cents} \div 5 \text{ yds.} = 15 \text{ cents per yd.}$$

are frankly advocated.

In each of the three chief lines in which arithmetic is actually used, business, engineering, and science, concrete multipliers are incessantly made use of. Only in the schoolroom is it insisted that the operators must be abstract. Unfortunately most teachers do not realize that the merchant thinks in cents per yard and dollars per bushel, as the railroad man does in ton-miles, car-miles, and miles per hour, as the engineer does in horse-power hours, miles per hour and so on indefinitely. The important point is that these are units, single definite ideas, though composite ones: cents per yard is neither cents nor yards, it is *cents-per-yard.* "A far truer history of this country could be written with the cost per ton-mile as a text than all the wearisome volumes of battles and wars, and the learned tomes of constitutional history."

Professor John Perry is quoted as saying: "If I were asked to multiply 2 tables by 3 chairs I would not refuse, I would say 6 chair-tables. But if I were asked to say what I mean by a chair-table I would refuse, because nobody has ever given a meaning to the term. But I do know that when this sort of thing comes into a Physical Problem we can always give a useful meaning."

Mensuration is the subject into which the use of the concrete multiplier and divisor should be first introduced. Rather than the usual circuitous forms of analysis, it is suggested that it would be better to explain, in introducing the definition of the unit, that the square on the blackboard is one foot by one foot and is called a square foot, and that, when it is written in the short way we write things in arithmetic

$$1 \text{ ft.} \times 1 \text{ ft.} = 1 \text{ sq. ft.}$$

[1] Popular Education, January, 1911.

Then that the rectangle is 1 ft. by 3 ft. and so its area is

1 ft. x 3 ft. = 3 sq. ft.

and finally, the child should at once write

7 in. x 6 in. = 42 sq. in., and so on.

Similarly in problems from business the form, "If 5 yards cost 75 cents, the *number* of cents per yard which it costs is the number of times 5 is contained in 75," should be discarded for

75 cents ÷ 5 yds. = 15 cents per yd.

The conclusion is reached that "the extension of the definitions of multiplication and division so as to include concrete multipliers and divisors is a perfectly legitimate process of reasoning. In fact, such an extension is already in general use in business, engineering, and science, the elementary schools being the only place where such an extension is forbidden. It is held that the method should be cautiously tried, that it would clarify and simplify the treatment of mensuration, and often be of advantage in the solution of applied problems taken from business, that it would aid in clearness and directness in attacking problems. It is further held to be important that teachers should understand these generalizations of the fundamental processes which are made use of by those who actually use arithmetic, in order not to repress spontaneity in the brighter pupils, and that such a broader view would lessen the at present altogether too prevalent use of set forms of analysis."

It is added that the primary object of the article is to bring out a discussion of the question.

MINERALS FROM ALASKA IN 1910.

The estimated value of the mineral productions from this territory in 1910 is $17,400,000; the value in 1909 was $21,146,423. Of this, the estimated value of the gold output in 1910 was $16,360,000; that of 1909, $20,371,078. The copper production in 1910 is estimated to have been 5,600,000 pounds, valued at about $740,000; that of 1909 was 4,124,705 pounds, valued at $536,211. The value of the other mineral products, including silver, lead, gypsum, marble, and coal, is estimated at $300,000—an increase over that of 1909.

The total value of the Alaska mineral production since 1880, when mining was begun, is, in round numbers, $186,000,000, of which $179,000,000 is represented by the value of the gold output.

CIVIC BIOLOGY IN THE HIGH SCHOOL.[1]

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Civic biology concerns itself with the welfare of the community, state, and nation. A course in civic biology should bring the student face to face with the biological problems of his community and point out their bearing on the larger problems of his state and nation. No one course in biology can be given in all the high schools of the country; each school must modify the course according to the flora, fauna, interests, and industries of its vicinity. Every teacher must base the necessary modifications of the course on the answer to the question, *What biological knowledge is essential to the community in order that its members may promote their health, and develop and conserve their natural resources?* It is obvious that in a rural community the course must needs savor strongly of agriculture; in the city it will deal with the problems of public health and civic improvement.

The course should begin in the fall and continue throughout the year. Indoor and outdoor laboratory exercises should characterize the work. The order in which the course should be developed is largely determined by nature; much of the work must be begun in the fall and left until spring for completion, while other parts of the courses are well adapted to winter study. The work outlined in the syllabus below is much too extensive for a year's course in any high school; all that is not vital to the interests of the community should be eliminated.

Birds. The student may be pleasantly introduced to civic biology through the study of birds. His attention is directed to the observation of birds in the field. He adds to the number of species with which he is already familiar; he studies their beauty of song and feather, their habits and, above all, he learns of the work they are doing to prevent insect depredation. The value and method of taming and domesticating important species are pointed out. Fall migration is noted and records of birds are kept to compare with those of the spring migration. Winter birds, their food, and habits are studied, together with methods for protecting them in the severe weather. Observations on nest building, feeding, and the care of the young round out the year's work.

[1]This paper was read at the Conference on Biological Instruction held in connection with the Second Decennial Celebration of Clark University, Sept. 8, 1909.

Plants of the Locality. While the student is studying birds in the field, his attention should be directed to the important plants of the locality. He should make the acquaintance of the native trees, shrubs, vines, and the noted wild flowers of literature, and learn concerning the general character of their habitats. He should, moreover, make a study of a number of water plants, learning in the course of his investigations, their use as food for aquatic animals, their work of purifying water and filling in lakes and ponds.

Insects. While the student is observing birds and their work, he should read a general account of insects, informing himself concerning their number, their power of reproduction, and the estimated loss the country as a whole is sustaining through their depredations. Attention then should be directed to the work of insects of the locality. Reports should be made of their work in garden, orchard, crop, and forest, together with losses sustained in the pupils' homes. The natural enemies of injurious insects should be learned, their work recorded, and methods of protecting and propagating them discussed and tried.

Beneficial insects should be recognized, and the character of their work known so that proper protection can be accorded them. With the study of the insects of the locality, and of the best methods to be employed for their extermination or protection, should come a more detailed study of special insects in the laboratory, the mosquito, cabbage butterfly, and ant, are especially good civic types.

Arachnids. The Arachnids lend themselves well to fall study. Harvestmen, "Daddy Longlegs," and spiders, may introduce the student to the order, and their structure, habits, and food may be compared with those of insects. The presence in the locality of parasitic Arachnida—viz.—spiders, harvest mites, itch mites, poultry ticks, and Texas fever ticks, should be determined, and the most effective means for their extermination learned.

Weeds, Medicinal and Poisonous Plants. The class may study this group of plants late in the fall. A list of the most noxious weeds should be learned and their life histories and habits studied. The characteristics which make plants weeds—great reproductive power, vitality, tenacity, adaptability—should be noted and compared with those of less aggressive food plants. The student learns that weeds damage the crop yield by robbing the plants of nourishment and light, and depreciates the value of real estate by offending the æsthetic sense. Much stress is laid upon weed

management. The practical method of dealing with weeds over large areas is based upon the life histories of the weeds themselves. Certain weeds follow certain crops and by proper rotation a system of weed management may be practiced which gives the weeds the least possible chance to grow.

The student learns the source of our crude drugs, of the experimental work of the Department of Agriculture in domesticating the wild drug plants that are in danger of extermination, and of its efforts in adapting foreign drug plants to our soil and climate. He is also taught to identify the poisonous plants of his locality, and gains a more general idea of their character and the damage they do, through Bulletin 86 of the United States Department of Agriculture.

Civic Improvement. In the field laboratory work, the attention of the student may be called to the shrubs, vines, and other plants that are available for decorative purposes. The idea that actual pictures may be made with these trees, vines, and shrubs is kept constantly before the pupil and good and bad examples of planting should be pointed out. While possibilities and plans for the improvement of his own home grounds are uppermost in the pupil's mind, his interest soon reaches out to his street, public parks, and buildings, and his watchword becomes that of the American Civic Association, "More Beautiful America."

Fishes. Fishes may be studied either in the spring or fall. Pupils should be encouraged to go fishing in the different bodies of water to stock their aquaria with the food and game fishes of the locality for early winter study. Thus the pupil learns at sight not only the names of the important fishes, but also many of their habits and habitats. The decrease of fishes in the locality is brought to his attention, and he learns of the need of the laws made for their protection. His interest in local fish problems extends to that of the state fishery and the United States Bureau of Fisheries and their work of propagating valuable species to restock depleted waters.

Mollusca. The study of mollusks may be left until spring or they may be collected in the fall and kept for early winter study. The student learns the species that are used for food, as the oyster, clam, and scallop, and the economic problems pertaining to their culture. He also becomes acquainted with the more common species of mollusks that are in his locality, the damage they do in gardens, and their work as scavengers in purifying water.

A fresh water mussel, a common pond snail and a land snail are studied in the laboratory.

Mammals. Mammals may be studied in the winter months. While the student is observing the mammals of the locality in the field, he may be reading and discussing the important problems relating to American mammals such as:

1. Extermination of predaceous forms, panthers, wolves, mink, and weasels.

2. Utilization as food of native wild animals during the early settlement of the country.

3. Trapping fur-bearing animals.

4. Efforts to prevent the total extermination of valuable species.

The enormous damage done by rats, and the part they play in carrying bubonic plague, make them an excellent mammalian type for study in a course in *civic biology.* Live rats may be studied in the laboratory and the student may practice the most effective plans for their extermination in his home. Thus with the combined efforts of the class, "the worst mammalian pest known to man" may be eradicated from the neighborhood.

Fungi. The greater part of the winter should be spent in studying mushrooms, moulds, yeasts, fungous diseases of plants, and bacteria. Mushrooms may be collected in the fall and moistened for winter study. The parts of a mushroom are learned and especial attention is paid to the identification of the poisonous and edible species. A study of the yeast of the common molds, of their economic importance, and their work in nature follows the work upon mushrooms. Parasitic fungi, those causing plant diseases, are collected in the fall and are studied after yeast and moulds. Their prevalence, damage to vegetation, and the best methods of control are discussed. After the student has worked with yeast, moulds and parasitic fungi, he can readily undertake the work on bacteria. He learns first hand the size of the plant, its power of reproduction, and the conditions best suited to its growth. He also learns the prevalence of bacteria in air, soil, and water, and the part that they are playing in the economy of nature the work of the beneficial species of bacteria and the sickness and death that follow parasitic forms are learned. The prevention of bacterial diseases and their cure are discussed at some length together with the necessary care of the body to ward off disease. The course is practical and the student coöperates with the physician and board of health in his work.

Animal Parasites. The harm done by the animal parasites in producing diseases should follow that of the parasitic bacteria. Malaria and yellow fever, their cause and prevention and the relation of their distribution to the mosquitoes are dwelt upon, together with hydrophobia and the various worms parasitic upon man.

Classification of Animals and Plants. In the early spring before field collecting can be done, the student may be taught to classify into the groups to which they belong the plants and animals with which he is familiar.

Apple Tree as a Type. A common yet important plant of the locality, such as an apple tree should be studied in all its bearings. Its environment, structure, its work as a living organism, and its relation to air, soil, and water should be understood. Observations on the fall and winter condition of the tree should be made, previous to the more detailed study in the spring.

Improvement of Cultivated Plants. After the student has learned the general activities of plant life he is able to understand and appreciate the improvement of plants brought about by breeding. He gets a glimpse of the plant-breeders at work improving the quality of cultivated crops; he learns also that wheat is made to yield as high as twenty-five per cent more to the acre, and that it costs no more to raise improved varieties than those that are not improved.

The loss sustained by planting crops that are not adapted to local conditions is brought to his notice, and he sees the significance of breeding plants that are adapted to the soil and climate of the different parts of the country—plants that are bred to withstand the severe climate of the North and the extreme drought of the semiarid region of the Northwest. Because he has learned the damage done to staple crops by fungi, he doubly appreciates the work of breeding disease-resistant varieties of plants.

The principles governing plant-breeding should be taught in a simple, concrete way. The student thinks of qualities that would improve a plant and searches for desired variations, which he could make the basis, through selection, of the new type.

Forestry. The forestry problems should be left until the student has gained some clear and definite ideas of Civic Biology. His work with trees—their power of reproduction, their means of dispersal, and their relation to soil, air, and water—makes the problem of trees growing together in a community, the forest, much more easily understood. He also learns the part that

trees and lumber play in a civilized country. He speculates on the rapid disappearance of the forest and its effect on floods and the industries of the country. He becomes interested in the conservation policies of his state and nation, and becomes a strong advocate of reforestration of all possible land in his locality.

Crustaceans. The student who lives inland may be introduced to the crustaceans through the crayfish. He studies it in the field and in the laboratory; he learns of its resemblance and relation to the lobster in form and habits before he takes up the economic study of the members of the group, viz; lobsters, shrimps, and crabs that are useful as food. He learns of the work of the national government in propagating and protecting the crustaceans by law. Small inconspicuous members of the group should be collected, and their value as food for young fishes discussed.

Amphibians. As soon as spring opens, the student should begin to collect the amphibians of the locality. They are kept in the laboratory under as normal conditions as possible, and their life history, habits, and activities studied. Feeding tests are made by the students to ascertain their value as insectivorous animals. A special study of the common toad is made.

Snakes. Snakes are best studied in the spring both for the reason that they hibernate early in the fall, and for the reason that the popular prejudice is so great that students would be unwilling to work with them at the beginning of the course. The student is taught how to recognize the poisonous from the nonpoisonous ones. The garter snake is made the subject of special study in the laboratory, and the fact should be brought out that, because so little is known about the food habits of snakes, their economic importance is not as yet settled.

Biological principles, problems, and laws should be taken up from a purely practical and elementary point of view relatively late in the course. The law of geometrical increase thus has many practical aspects in regard to extermination of injurious species, protection of valuable species and propagation of new varieties. Problems in this field definitely stated, figured out and carefully discussed, viz; problems dealing with the rat, English sparrow, mosquito, house fly, San Jose scale, various bacteria, weed seeds, valuable seeds, and buds—must help toward forming a social organization fit to deal with them.

Of all human history that of the gradual growth of science—the struggle for truth, the slow and patient march of discovery—

is far and away the most interesting. It is thus recognized that no course in any science is fit to survive which neglects the historical side. Can we throw our pupils into this stream of historical effort and teach them to swim in it, make them feel that they are a part of it, make them feel that the story is not all told yet, that, if they are willing to work hard enough, they may add something to it? These are large questions and I cannot assume to have answered them; but suppose we let the class work up and write out its own history of biology. We might post a list of the more important names from Aristotle down to the present, supplying perhaps, the dates to give a notion of sequence, and just a word or sentence to indicate the contribution or the field in which the man worked; then let each pupil choose some one master and let him study his life and work and write a brief account of what he finds. These studies may be arranged for early in the year and then the papers may be read by the pupils along toward the end, so that each may have time to "make a little journey," if you please, to the home of his master, and get acquainted with him as he lived and worked. This, of course, is merely a suggestion, which each must work out for himself.

Closely related to the historical growth of biology, and carrying more definite civic values is the concluding feature of the proposed course, viz; a definite series of lessons on the national and civic organization of biological research and instruction. We may entitle it, "American Naturalists at Work." The work of the different men may be assigned, as suggested above, to the students and they can write about them and report to the class. They should have the pictures of the naturalists on their expeditions, in their camps, in their laboratories and workshops. They should be taught the organization of the Department of Agriculture, especially that of the Biological Survey; and of the United States Fish Commission. We must introduce our youth to the American Ornithologists' Union and the National Association of Audubon Societies; to the various entomological, mycological, and bacteriological societies; to the American Nature Study Society, the Agassiz Associations; and above all, we must bring every boy and girl to want at least to join the American Health League. In a word, this work must focus the whole course, bring the young men and young women to feel their own vital relations and responsibilities to our national effort, to the end that each may seek out and find, the part to which he is best fitted, and begin to do his part for the good of the whole.

INSTRUCTION IN SOCIAL HYGIENE IN THE PUBLIC SCHOOLS.[1]

By C. F. Hodge, Ph.D.
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"Better a year too early than an hour too late," is a German watchword which must appeal to every American parent and teacher. We must keep the germ plasm on the up grade, free from taint, at any cost. This is the supreme law of human evolution. To pass on the torch of life, not only undimmed, but ever brighter from generation to generation, is the highest service which parents of any generation can possibly render. This is what evolution means, and in its faith we may share the hope of Professor Bunge when he says: "The time will come when a race of men shall rule the earth as much superior to ourselves as we are now superior to amebas."

I need waste no words in depicting our present-day "Great Black Plague" and the literally sickening conditions which menace the evolution of our race. As the dawn of scientific light breaks on this field, we see that we have gone far astray, even to falling into the very slough of despond itself. Our home, church, and school education have proved pitifully inadequate. We cannot hope to effectively reach parents—heterogeneous, busy, untaught, scattered, often foreign, immoral sometimes themselves. Hence, our main hope is through the schools, to save the young, at least from falling into the mire or becoming contaminated through ignorance. "Wait fifteen years," said a leading educator, "before you try to put sex hygiene into the public schools." This was a year ago. The same man now says: "We have moved rapidly in this direction. The people see the need and are ready now." But this most delicate teaching must be done right, must be true to the highest and best instincts of the child, or more harm than good may result. The teacher must work with consummate tact and should first of all receive the cordial support and approval of parents, especially the leaders in the neighborhood before undertaking the task. If these parents cannot be heartily enlisted in the teacher's plan and methods, I see no possibility of going further with this instruction.

Sex is, in fact and of right, the "ruling passion," and all recent studies, both here and abroad, unite in emphasizing the extremely

[1]Read at the thirty-fifth meeting of the American Academy of Medicine, St. Louis, June 6, 1910. Reprinted, by permission, from the Bulletin of the American Academy of Medicine, October, 1910, pp. 506-517, with slight revision by the author.

early development of sex interests in children. They are growing more and more precocious with the increase of city life, and too many parents are rudely shocked to find that what they had supposed to be the innocence of infancy has already been befouled, and that a filthy philosophy of sex has been formed, hard or impossible to eradicate, which may cause years of distress and which may make wholesome instruction well-nigh impossible. Every consideration demands that we forestall this calamity with really true-to-nature instruction, wholesome, adequate, intelligent, and inspiring.

The scarecrow instruction of a generation ago confined entirely to the home was directed mainly against the evils of self-abuse. A measure of this may be necessary in special cases still, but in the main it is negative, depressing, uninspiring, and by overemphasis is thought by many experts now to have tended to drive unduly from the lesser Scylla of self-indulgence to the worse Charbydis of promiscuous intercourse. We do not need to steer anywhere near either of these dangers. All modern views urge us to make the instruction thoroughly positive. Make it full of enthusiasm for sound health, strength, vigor of brain and of every muscle and organ. Fill the child's life so full of good work and play, especially out-of-doors, along the line of nature study and manual training, athletic sports, gardening, camping, etc., that he will not have energy or time to waste. Dr. W. S. Hall's suggestion to saturate the children with hero lore indicates the very best tendency in modern education to long-circuit sex and transmute and sublimate its emotions into motive force for worthy achievement and best possible development of self. It is hard to see why sex should outcrop so early if this is not its true function, to stimulate growth into perfect manhood and womanhood. If it is not controlled, if it is allowed to dribble away in self-indulgence and mere satisfaction of a momentary passion, normal growth may be arrested and the full stature of psychic parenthood be never attained. When we contrast a person arrested in a life of mere sensual pleasure, who does not have even the desire for offspring, with one whose sex impulses have been held to the channel of personal achievement and the rearing of a family, we see the great need of control at this crisis of childhood.

Lead the child to control and sublimate sex—this must be the task of the teacher and parent, while the child is as completely unconscious of himself as possible.

Positive inspiration, too, for personal cleanness, in the full light of modern knowledge of the causes of disease, must be our main dependence in protecting school children from the social as well as from all other preventable infections. Little more than five years ago it was considered preposterous to try to teach anything about bacteria and the germs of disease in the elementary schools. Now we have "Primers of Sanitation," "Primers of Preventable Diseases," and good health series of texts in the hands of the children and fairly adequate source books and health board reports and leaflets on the teacher's desk. This has come about from a clear realization that the very edge of the human struggle for existence lies between man and these minute and well-nigh omnipotent organisms. The theory is as simple as ordinary gardening—the planting of cabbage seed and the rearing of cabbage therefrom. So seeds of disease are planted from sick to well by commonly simple and easily understood means of contact and transference. By placing all the emphasis on cleanness of person and habits, on health and vigor, instead of morbid fears, as some have thought, we get even in young children the full joy of assured health and safety, and not a single bad result.

All this literature shuns absolutely any mention of the specific social infections, on the ground that this whole subject is too filthy and sad and suggestive of evil to place before young children. In the now three years of medical inspection of the Worcester Schools not a single case of accidental infection of either gonorrhea or syphilis has been discovered in the children of either grammar or high schools. This might be taken to indicate that the school machinery already has the matter effectively in hand, but education has not done its full duty unless it prevents infection, both natural and accidental, at the critical period just after leaving school. The great majority of infections occur at this time, and if the school is to aid in staying the present flood of youthful disease and misery, it clearly must furnish specific and adequate instruction before the children leave it. The body of this instruction has not been worked out, and I can only suggest tentatively in brief outline its most essential points. It must necessarily fall into two courses of lessons, one intended for children toward the end of their grammar school, the other as an integral part of their high school course in biology or physiology and hygiene. From all I am able to gather from the literature of the subject, this will require years of careful elaboration by the best equipped and most intelligent teachers.

For instruction of grade children the lessons should consist for gonorrhea in a plain statement of the biology of the germ, its natural method of distribution through promiscuous intercourse and the course of the disease in the male and female. It should also caution against accidental infection of eyes and genitals by means of possible contact with the germs in public toilets, towels, bathing suits, and the like. The very serious character of the disease, its menace to possible future parenthood, its complications and frequent late effects, and the difficulty in freeing the body from the germs should be reasonably emphasized.

Similar lessons should follow on syphilis, but here much more detailed instruction must be given in the matter of avoiding innocent infections. Recent statistics point to the fact that as high as 38 per cent of all infections in this country are innocent. The long course of severe medical treatment from four to five years necessary to free the system from the germs, leaving it even after this with debilitated blood vessels and often wrecked nervous system, the great danger to offspring, even after "cure," ought to be presented in plain and simple manner, but it should make it impossible for any child to become infected either ignorantly, carelessly, or viciously.

For both of these infections it must be clearly taught that to communicate them is a crime not a whit short of deliberate murder.

Taken in connection with lessons on the prevention of other contagious diseases, and done tactfully, there need be no danger of calling undue attention to sex itself, the emphasis being all laid on health and purity. A good deal of expert testimony from the side of teachers who have been studying the problem indicates the wisdom of dealing with the boys and girls separately in these lessons, mainly because the instruction needs to be radically different for the two sexes. The girls should have the side of normal healthy function emphasized and be given a minimum on the side of disease. At any rate, it is held that the appeal to healthy, normal motherhood is all-sufficient with girls and that if only they are given the precautions and relations correctly they will strictly avoid anything which is likely to endanger this function. They should be shielded, especially girls under sixteen, from too specific knowledge of the abnormal side for fear of morbid tendencies, shock, or arrests in normal development both bodily and psychic.

For the boys the side of disease should be painted truthfully and if so, it needs no exaggeration to constitute this a most effective safeguard against taking any risk of infection. They must be clearly taught that no manner of promiscuous indulgence is or ever can be safe, and that the only safe rule for perpetuation of a clean and vigorous race is that of the single standard of morality. We must make it impossible for any young man to say, as so many are saying now: "If I had ever been given ten minutes plain instruction in these matters, it would have saved me years of disease and suffering." My experience along with that of many others indicates, too, that we must clearly forewarn the boys against the pitfall of the vampire. It is probably safe to say that no statistics exist in the matter, but my own data gathered during a pretty active lifetime indicate that first experiences of young men are in a vast majority of cases vampire seductions. And I feel that any young man who starts out into life for himself, away from home into business or to college, without a clear notion of her wiles is starting out on thin ice.

Boys, too, need more specific instruction about hygienic care of their organs, probably, than girls. They should early learn that these must be kept clean—cleaner than any other part of the body—and if sensitive and troublesome, they should be taught the value of the cold bath with ice water, if necessary. They must be taught that their first concern of life at the period of beginning adolescence is to work, both mentally and physically, to exercise and play, so hard that control of sex becomes easy, perfect, habitual, automatic. They should be made to realize early, anyway by fourteen or fifteen, that the control of sex is literally the crucial test of character, and that the sublimation of this energy into growth, stamina, strength, and achievement is sometimes eventually to decide whether their lives become really worth living or not. If they are ever to do anything great or heroic, or even hope to be fit to do this when the occasion offers, they will need this store of vitality and strength and the power of perfect control, as they will need life itself. I cannot possibly speak as plainly thus to a mixed audience as I do to a gang or a class of boys, but here is a point where a boy can be touched to the quick and straightened out for life, if there is any life in him.

About this age, too—fourteen to sixteen—the boys must learn that erections and emissions during sleep are perfectly normal, healthy functions—nature's safety valve; and that unless com-

plicated by consciously vicious practices, do not in any sense drain the energies of the body. This "young man's problem" is the very most intensely vital of his whole life, and its healthy and sane solution lies fundamental to practically all his other serious problems. Perfect power of control before marriage means power of control after marriage, and thus may save this, which should be the happiest and noblest relation in life, from becoming the beastly slough of despond so much of it now is. Happily the time is now well past when the older man, or even the family physician, can advise the young man to seek temporary relief in the solution of his problem by the soul-degrading and body-contaminating contact with the strumpet or prostitute. All young men must learn early that to begin before marriage is to give up the fight in defeat, that the time to control the conflagration is before the fire is kindled. This comes as the most insistent, and almost irresistible temptation of early adolescence. "*Just once—to see if I really am a man.*" Now let the vampire come and the boy that resists has stuff in him that is worth while. "This cockatrice is soonest crushed in the shell; but if it grows, it turns to a serpent, and a dragon, and a devil" (Taylor).

I would not give the impression that for a man, recovery of some degree of self-respect is not possible. If disease is not contracted this may be won, but it generally is not; and if it is the struggle is tenfold harder, with the shining goal of undimmed purity irreparably lost. The time will come—and every boy should be told this, better a year too early than an hour too late—when he may be in honor bound to make a clean breast of everything, if he have a spark of honor in him. And when this time does come, the young man who can offer purity to purity would not have it otherwise for the gold of Midas. And the time must soon come, too, if it be not already here, when this will be the only perfect and acceptable offering on the altar of marriage, higher, as is life itself, beyond all estimate, than millions or titles. Boys need to be told this, sharp and clear, and I have yet to see a pure boy who does not rise to this ideal as a soul to his own star.

In the early instruction of girls the writer has had little experience outside his own family, and he must leave this side to others. But here purity is taken for granted and calls for no argument. It does seem, however, that the old-fashioned instruction as to the signs of virginity ought not to be allowed to lapse completely. True, these are not absolutely clear. The hymen may be ruptured accidentally or carelessly. The nipples

may enlarge and the areolae may darken in exceptional cases. At any rate, it can be only wholesome to have every young girl realize that guilt may be written indelibly on her body and that this may sometime prove more terrible to her than a death sentence. Recent studies seem to indicate that loose girls are quite generally defective, or feeble-minded or half-witted. And it does seem that this must be the case with every girl who has not native sense enough to realize the life-value of her purity. If it became universally understood that any lack of control implied feeble-mindedness, the mere suggestion would act as a powerful balancer to the sexually unstable. Of course the proper instruction with regard to the monthly function and cleanliness is taken for granted.

Young people should also be clearly and early taught that while control may be difficult under improper conditions, their first plain duty is always to control the conditions and keep them wholesome and free from possibilities of temptation. Here even slight alcoholic intoxication is the most dangerous pitfall in the way of a young man. In this condition what would normally shock and disgust may appeal to him as attractive.

The problem of an adequate course in sex hygiene for high school boy and girl requires the space of a separate paper, and college instruction might well be the subject of another. The briefest possible sketch of my plan for the high school must suffice for the present.

The high school age is the time above all others for fixing ideals. It is then that dreams of perfection, altruism, and ideality bloom if ever in the child's life. In even the last year of the grammar school any specific instruction is probably apt to force somewhat the budding consciousness. With the definite change to the high school and in connection with a well-ordered course in biology which is arranged to prepare seriously for citizenship, along in the course toward spring of the first year is, in my opinion, the strategic time. The pupils will have studied, seen, handled and drawn the fungi that attack plants, they will have made cultures of non-pathogenic bacteria and been given the technique of sterilization and all the rationale of bacteriological cleanliness. They will have been given a sharp, clear course in the biological relations of the few common germs that cause human disease—and in methods which must be known and used to prevent their spread from sick to well—such as typhoid, tuberculosis, diphtheria, pneumonia, small-pox, grippe, measles, malaria—and we must

now add definitely ***gonorrhea*** and ***syphilis***. The work indicated for the grades should be given a thorough review and should be filled out and completed especially along the lines of effectual prevention. A small but rapidly growing percentage go on into the high school and these should go out from it equipped to leaven the whole lump of our industrial life with saving knowledge of the truth.

Again in the detailed study of processes of fertilization in plants and in the dissections—or if that is too strongly objected to, in demonstrations—of the sex organs of lower animals, the students must first of all come out into an honest respect for the importance and dignity of sex. And as a teacher I cannot see how we can possibly hope to have a notion of the problem of the sex infections vitally developed without a clear understanding of the organs and their anatomical relations. One good look at the female organs—the vagina, with its inaccessible openings to the bladder, ureters and kidneys, with the uterus and Fallopian tubes opening directly into the peritoneal cavity—simply means that a female once infected with gonococci is to the end of her life infected, unless the organs are removed surgically. A study of the male organs—the urethra, ureters, the complicated glands and vesicles, the minute, long and tortuous, spermatic ducts—show, as no amount of lectures or text ever can, how likely sterility may result from infection and how impossible it is to be sure that an infected male is ever entirely freed from the germs. Gonorrheal stricture has been known to develop and virulent infection to be transmitted to an innocent wife and babe as long as thirty years after a supposedly complete "cure." The way in which the eyes of the new-born are infected and the fact that about 80 per cent of common operations on women, which, it is generally recognized now, must result in sterility, should be carefully treated.

In tracing the course of syphilis we must emphasize the fact that it is a through and through systemic infection. The entire body is diseased. As one writer puts it: If a man contracts syphilis, he lives a syphilitic, his children will be syphilitic, he dies a syphilitic, his corpse is syphilitic, and his ghost on the judgment day will be syphilitic. This renders it much more likely to be communicated than gonorrhea. Anything which touches a syphilitic or which a syphilitic touches is apt to carry the infection—all sorts of eating and drinking utensils, towels, handkerchiefs and toilet articles, kissing, and even kissing the

Bible in taking an oath. The cough spray of a syphilitic may be more dangerous than the discharge of a gun into a person's face. The public drinking-cup is banished by law in some states, and should be in all, because so often the means of transmitting the loathsome germs from the mouths of the sick to the well.

It should be clearly taught that syphilis is, by its hereditary transmission, the cause of more racial deterioration than all other diseases combined. It is the one disease in which the germs pass directly from either parent to the offspring with generally fatal but often worse than fatal effect. It may thus be directly transmitted even twenty years after a supposedly complete "cure." General debility, an impaired nervous system manifesting itself in low efficiency and criminality and often in mental or moral idiocy or imbecility are the frequent results. The blood vessels are often permanently injured, and, even after "cure," fail to nourish the organs properly, causing especially grave degenerations of the brain and spinal cord. These effects cannot be predicted with certainty and often cause later in life apoplexies, paralyses, and insanity.

Recently exploited cures for syphillis, notably Ehrlich's 606, may afford some relief and hope to afflicted individuals; but it will require a generation to demonstrate that they are free from these belated degenerations and after effects. That the germ cells, too, may not be permanently impaired must require at least twenty years to disprove.

Looking at the problem from a purely biological and eugenic point of view it is hard to understand how medical authorities can ever advise the marriage of a syphilitic with an uncontaminated mate. It is high time that society faced this problem resolutely and adopted measures which shall effectively protect itself from the increasing burdens of the syphilitic defectives and degenerates. We are beginning to insist upon "pure foods" and "pure drugs." Of infinitely more vital moment are *"pure mates."* It would be only reasonable, if society should insist that syphilitics should be sterilized and be permitted to marry only syphilitics. And also that those who have been infected with gonorrhea should be permitted to marry only with gonorrheics. These principles generally disseminated and understood must put a speedy end to the careless, jocular lying about such infections being no worse than colds in the head.

We come finally to the *crux* of the whole matter. Can education do anything to elevate and purify our ideals of marriage?

We are living in an age of ignorance and frivolity which too often results in marriage for the fun of it. Authorities high in social psychology even go so far as to openly justify this position on the ground that man has evolved beyond and higher—rather than lower—than the brutes, in regard to what is legitimate indulgence.

I cannot help feeling, and feeling strongly, that this is the most foully rotten thing in all modern philosophies of life. If this position is the true one, then the more "fun," the higher the evolution; and if marriage is so trifling a matter, why not dispense with the formalities and restrictions? So here we have the ultimate ground for unrestricted, careless, easy, unwholesome marriage, with its consequent relief in divorce. And, as the running sore of this abnormal condition we have promiscuity and prostitution. These latter are but the incidental ills resulting from a bad philosophy of sex and do not tend to thwart and corrupt the great stream of racial evolution as do anti-biological practices in the marriage relation itself.

My experiences with college men encourages me to believe that in connection with thoroughly sound eugenic instruction we can straighten them out for life on this point so that in the marriage relation they shall feel that they "must be sure to observe the order of nature, and the ends of God" (Taylor). Place in the balance, on the one side, any amount of selfish indulgence; on the other, blooming offspring and a man's position in the evolution of the race; it requires scarcely ordinary intelligence to see which beam kicks up. This becomes, too, the young man's best inspiration to vigorous health, good work and avoidance of bad habits. We need sublimation of sex quite as much after marriage as before it.

As woman is winning her place in our social economy, we may confidently expect the best help from her in solving these eugenic problems. The first "right," which she will be heartily accorded as soon as she claims it in the name of a higher and better-born humanity, is absolute control of her own person. She alone can feel when her delicate organism is ready to meet this supreme call of life, and any interferences with these impulses and instincts must come to be recognized as the unbiological crime that it is. Young men see the reasonableness of this view, at any rate, if they have been given some insight into the biological side of the problem, and readily adopt it as their ideal. The time cannot come too soon when this right shall be accorded to

every woman as a universally acknowledged part of her marriage contract. As the poet Burns puts it: "Your slightest wish shall be to me as a sacred command."

A vision of what we may hope will be the perfect marriage of the future is given us by Marguerite Ogden Bigelow in the *Independent* for August 18, 1910—"A Bride's Psalm of Joy."

Right here in these social problems is the Augean stable of modern life, and the strenuous labors of many a Hercules will be required to cleanse it. Upon this cleansing, more than upon any other one thing, will depend the vigor and health, the stamina and perpetuity of the nation and the higher evolution of our race. Clean, true living is the price we must pay for racial improvement, and education, clear and strong and universal, is our only hope. Only in the clear light of truth can we see what clean, true living is. Let there be light, and let it come to our youth—"better a year too early than an hour too late."

PETROLEUM PRODUCED IN THE UNITED STATES IN 1910.

The year 1910 has been sufficiently eventful in the development of new oil supplies in the United States to more than keep up the country's phenomenal production of the last three years. In fact, the production increased to over 200,000,000 barrels, which is two thirds of the world's production and several million barrels more than the whole world produced seven years ago.

The most surprising developments of the year were in the Sunset-Midway district of California. The developments in the Caddo field of Louisiana also entirely changed the position of that field. Connected with the developments in both these fields were important changes in the conditions of marketing the oil.

The production of oil in the United States in 1910, as reported by D. T. Day, of the United States Geological Survey, was between 200,000,000 and 208,000,000 barrels, approximately, as follows:

	BARRELS.
Illinois	32,000,000
Appalachian and Lima-Indiana fields	32,000,000
Gulf and Caddo fields	14,000,000
Mid-Continent and Rocky Mountain fields	53,000,000
California	73,000,000
	204,000,000

TESTING THE RESULTS OF THE TEACHING OF SCIENCE.[1]

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The topic which I am asked to discuss is one of enormous complexity. The changes in human beings which result from the teaching of science in schools are real, are measurable, and will some day be defined in units of amount as we now define changes in the rate of a moving body or in the density of a gas. But they include thousands of different elements; they vary with every individual; some of them can be demonstrated only long after school is completed; and at present units and scales in which to state changes in knowledge, power, interests, habits and ideals are mostly matters of faith. An adequate measurement of the changes wrought in one class by one course in physics would be a task comparable to a geological survey of a state or an analysis of all the materials in this building.

I must also at once confess that I cannot bring you the results of specific investigations of educational achievement in science, but only such suggestions as general experience in measuring human faculty and various educational products can provide.

These suggestions fall naturally into two divisions according as one searches for means of measuring the specific information, skill, interests and habits added by courses in science, or the more general changes in total mental make-up—in, for instance, open-mindedness, accuracy, zest for verification and the like.

The specific changes are, of course, the easier to measure. Indeed, my first suggestion is that we make scientific use of the measurements that we already make. For example, the regular school examinations are, or should be, careful scientific measures of important changes in our pupils. If we would test our classes with the examinations set by other teachers, have the pupils' work graded by other teachers, and print questions, work and grades, we should be making a start toward a real measurement of educational achievement. If examinations are worth giving at all, they are worth giving, at least occasionally, in such a way as to measure not only how well a pupil has satisfied some particular person, but also what he really is or knows or can do in certain special fields.

We need thousands of significant questions, in each science,

[1]Read before the American Federation of Teachers of the Mathematical and the Natural Sciences, at the Minneapolis meeting, Wednesday, Dec. 28, 1910.

thousands of "originals" in physics, chemistry and biology like the originals of geometry; and above all we need to have thousands of classes tested by outside examiners, as has been done in arithmetic, spelling, handwriting, and geography by Rice, Cornman, Stone, Earhart, and Thorndike. If an examination, instead of being a hasty, subjective selection of questions, graded still more personally (and alas, how hastily), were made a serious educational measurement, the examination papers of a year would alone give us a large start toward knowledge of what science teaching actually does.

Knowledge may, however, be measured more conveniently than by the examination of notebooks, essays, or replies to questions of the ordinary sort. These have the merit of adequacy and richness, but the defects of measuring too many things at once and too indefinitely. Greater uniformity in the use of the test, quickness in scoring it, and freedom from ambiguity in the numerical value assigned can be secured by the exercise of enough ingenuity. I will mention two tests as samples of the many that are possible. The first is an adaptation of a test, devised by Ebbinghaus to measure mental efficiency in general, in filling in words omitted from a passage. From even the hastily devised sample presented here it will be seen that this form of test is scored with reasonable ease. The speed of an individual in selecting words to fill the gaps and the appropriateness of his selections together measure his knowledge. The former is scored with no effort at all and the latter with far less effort than is required to evaluate answers to questions, essays or experimental work. The paragraphs and omissions therefrom should be arranged with care and improved after trial, but it may be of interest to some of you to compare the ratings obtained in six or eight tests of five minutes each like the following:

A body changing its position in space moves in a certain ....at a certain ....A............in the .... called acceleration. To change either the .... or the ....of a moving .... requires .... Suppose a pound of lead to be held at rest 500 feet above the surface of the ocean by a string and the string to be cut. The body will .... toward the .... of the ...., beginning to .... with a .... of just barely over .... and reaching at the end of one

second a .... of .... feet from where it started. In one second the .... will have .... from .... to .... feet per ....

The second is a very simple development of so-called association tests which I have used with good success in regular examinations in psychology for a number of years. It needs no explanation other than a sample.

Write after each of these words some fact which it suggests to you.

acceleration	gravity	current	lever
density	expansion	elastic	inclined

As useful means of measuring the interests aroused by the study of science, I suggest records of the books taken from public libraries, of the periodicals chosen in public reading-rooms, of the collections gathered and objects constructed by pupils, and a modified form of the test just described, the given words being much less easily provocative of thoughts about facts of science, and being mixed, if necessary, with words that would call up facts of science only in a person absorbed by scientific interests: The sample I give is left without such padding for disguise.

Write after each of these words some fact which it suggests to you.

work	time	wave	square
positive	light	level	change
water	rate	pull	book
mass	study	transform	gas
long	contract	heat	law

This latter test of interest should be varied using pictures of, say, a man rolling a barrel up a board into a wagon, a lightning flash in the sky, an ordinary balance scale, and the like, with a similar mixture of "innocent" pictures. Besides words and pictures, actual or described events can be used. If such association tests are to be used to measure interest, they should not be used previously in the form calling definitely for facts about science. These tests of interest may be used to measure both

special interests in particular sciences and general interests, as in fact rather than fiction, knowledge rather than opinion, or verification rather than dispute.

Of other means of measuring the general changes wrought by the study of science I will mention only two. The first concerns the power to utilize experience well in thought.

What is needed for this purpose is a series of problems or tasks, relative success with which depends as much as possible upon having power to use experience and as little as possible upon having had certain particular experiences. For example, relative success with the problem, "Which is heavier, a pint of cream or a pint of milk?" is determined largely by ability to select in thought the essential fact that cream rises and to infer its obvious consequence. The data themselves are possessed adequately by all, or nearly all, pupils alike.

To get such problems we wrote some time ago to one hundred teachers of science, half in universities and colleges, and half in secondary schools. I quote some of them:

Rain drops are coming straight down. Will a car standing still or one moving rapidly receive in one minute the greater number of drops on its roof and sides?

Is air drawn up a hot chimney or is it pushed up?

Since it is possible for a person to float in water why is it possible for him to sink?

A cylinder and a cone equal in base and in altitude rest on a plane surface. Which is harder to tip over?

A magnet attracts two iron nails. If the magnet is removed will the nails attract each other?

Is it harder to keep your hands clean in the winter than in the summer? Why?

How many surfaces, corners, and edges has a cube?

Which has the greater surface—a cube 10 inches on edge or a sphere 10 inches in diameter.

What is the largest mammal in the world?

Does an iron ball weigh more when it is hot than when it is cold?

If a bottle of gas which is lighter than air be placed with its open mouth upward, will the gas escape from the bottle or will the heavier air press the gas back into the bottle?

Is an incandescent lamp filament on fire?

Will a ship that will just barely float in the ocean, float on Lake Erie?

Will a pound of popcorn gain or lose weight or stay the same after it has been popped?

The second means of measuring changes in general power to think is an adaptation of one devised by Professor R. S. Woodworth, in which the pupil picks out from such a series as that below, the statements that are logically absurd, not possibly true. It will be seen that statements could be chosen which would test the power of analysis and of thinking things together in any field of science from the most specialized to the most universal. Following is an example of this form of test.

Put a mark in the margin opposite each of the following sentences which is absurd:

Though armed only with his little dagger, he brought down his assailant with a single shot.

Silently the assembly listened to the orator addressing them.

While walking backwards he struck his forehead against a wall and was knocked insensible.

I saw his boat cleaving the water like a swan.

Having reached the goal, I looked back and saw my opponents still running in the distance.

Offended by his obstinate silence, she refused to listen to him further.

The one-armed cripple was attacked by a dog which seized his wrist, but he pushed it off with the other hand.

With his sword he pierced his adversary, who fell dead.

While threading my way through the crowd, I came suddenly upon an old friend.

The storm which began yesterday morning has continued without intermission for three days.

The dogs pursued the stag through flower gardens in full bloom.

That day we saw several ice-bergs which had been entirely melted by the warmth of the Gulf Stream.

While sharpening his three-bladed knife, my cousin cut his middle finger.

Our horse grew so tired that finally we were compelled to walk up all the hills.

The red-haired girl standing in the corner is taller than any of her older brothers.

A bricklayer fell from a new building quite near our house, and broke both his legs.

The hands of the clock were set back, so that the meeting was sure to close before sunset.

Many a sailor has returned from a long voyage to find his home deserted and his wife a widow.

The two towns were separated only by a narrow stream which was frozen over all winter.

The great advantage of these means of measuring intellectual ability lies in their rapidity and objectivity. If well devised, only two answers are possible, the pupil is measured easily, rapidly, and independently of subjective factors, and his condition is defined in terms of a simple numerical value.

There is no time for me to discuss methods of making, recording and utilizing these or the hundreds of other equally worthy measurements of educational achievement, that is, of changes produced or prevented in human nature. Nor is this a proper occasion to outline the precautions that are required by the complexity and variability of facts of intellect and character and the absence of well-defined scales with equal units and known zero points, in which to measure facts of intellect and character. For our present purpose it is enough to know that, in spite of difficulties, the measurements can be made, and that a man of science can, if he will, be as scientific in thinking about human beings and their control by education, as in thinking about any fact of nature.

METRIC ADOPTIONS.

Through the Hon. Sec'y of "The Decimal Association" of England the editor is informed that Siam has decided to introduce the metric system of weights and measures without delay.

Also a metric bill is being considered for the governent of Malta, and has already passed two readings in the council of government. When it shall have passed the third reading, as it undoubtedly will, and is signed, two new countries will thus have arrayed themselves on the side of progress, with the constantly growing number of reformers.

NATURE-STUDY TEACHING.

By C. F. Curtis Riley,
Head of the Department of Biology, State Normal School, Mankato, Minnesota.

We may theorize and speculate just as much as we please regarding the "why" and the "wherefore" of nature-study and accomplish but little of material value. There seems to be an idea prevalent in some quarters that we are all "up in the air" on this subject. The writer believes that some workers who have shown considerable interest in nature-study do take this point of view—if it is a point of view; but he is inclined to believe that this is due in great part—there may be other reasons—to the fact that these men are not actively engaged in nature-study teaching.[1] Not infrequently the college and university instructor is apt to belong to this group; and yet on the other hand the writer wishes to state clearly that some of the sanest and most workable ideas regarding nature-study teaching have been advanced by men in institutions of this class.

Some of us do not wish to be classified with this "up in the air" group, for our ideas regarding the teaching of nature-study, in part at least, have become more or less clarified and definite. The writer, in a measure, can appreciate this position, yet he believes that we should not permit it to concern us too much; but should continue to put into practice the ideas that we already possess. The writer's work in nature-study has been chiefly with students of two classes—those in normal schools preparing to become teachers in the graded schools and in the high schools, and summer work with those of college grade. It is regarding some of the work with students of the former class to which he wishes to draw attention.

A large proportion of the nature-study work that is being done is for the purpose of preparing teachers to present this subject in the graded schools. At the present time and also for many years to come, the writer believes the subject matter should be drawn from the natural sciences—the biological sciences. Therefore the teacher of nature-study in the normal school or teachers' college should be a trained biologist with a liking for animal and plant behavior, and also some knowledge of agriculture and physiography. By this the writer does not wish to be

[1]The term "nature-study teaching" is used here in the sense that it is used in the normal school.

understood as stating that chemical, physical, geological, and even astronomical topics are not integral parts of nature-study; but he is convinced that the great majority of graded schools are not prepared for such work at present. It is necessary and pertinent that such topics shall be presented by the instructor in nature-study to his prospective teachers as can be put to practical use in their own schoolrooms. The writer is thoroughly convinced that much of the failure of nature-study as a part of the graded school curriculum can be attributed to the presentation of this multiplicity of subjects and to the fact that some of the instructors under whom the graded school teachers have studied were physicists and chemists instead of biologists. The writer feels assured that many instructors in nature-study will agree with him in respect to the two last statements. The writer does not assume this viewpoint because he is a biologist, for during his undergraduate work in college he was much interested in chemistry, physics, and geology, and has given instruction in these sciences both in the high school and the normal school and therefore fully realizes their importance.

Under present conditions in the graded schools the time element is an important factor in nature-study teaching and the writer believes that it is much more practical and feasible to teach the subject through the agency of the biological sciences than it is to present the subject from the physical and chemical point of view. Furthermore the facilities—suitable conditions and apparatus—are not to be obtained for such work in the graded schools. Much of the work of this nature that the writer has observed has shown itself to be practically worthless; in a great measure it has been time wasted so far as any return to the pupils is concerned. On the other hand, much valuable work can be accomplished, and is being accomplished, in nature-study from the biological viewpoint. Such study can be well carried on in the average *well-lighted* schoolroom. Comparatively little additional expense needs to be incurred, as no special apparatus is required for much of the work.

A school museum is one of the most helpful factors in nature-study teaching, but very few of the graded schools possess one. In large cities this problem can be solved, partially at least, by making use of the municipal museums. This can be done by taking the children after school hours or on Saturdays. But there are thousands of graded schools in the small towns and villages —not forgetting the country schools—where there are no mu-

seums. Thus much depends upon the teacher. If she is adaptable this lack can be largely provided for. She can in a few years build up a school museum—or at least a museum for her own pupils. Much material can be collected at no expense except that of time and labor. Insect collections can be obtained; nests of the mud dauber and paper wasp are easily collected; pupæ, chrysalides and insect cocoons are frequently to be found; crayfishes, frogs, toads, fishes, aquatic insects and larvæ may be kept permanently in formalin—an inexpensive preservative. Mason fruit jars can be used as containers for this purpose. Buds, stems, and mosses are readily preserved in alcohol, or formalin; dry seeds of many kinds may be kept indefinitely in small boxes, more satisfactorily in mason jars. Flowers, plants, and leaves may be pressed to form a small school herbarium. The uses of such a school museum may be further augmented by the purchase of a few inexpensive tanks in which fishes, frogs, crayfishes, and aquatic larvæ may be kept alive in the schoolroom—as children are vividly interested in living things. Many of the hardier plants may be kept in window boxes, so that simple botanical facts may be learned from living things. Beside this in the spring and autumn small school gardens can be prepared. Field trips can be made either after school has been dismissed, or on Saturdays, if there is not sufficient time during regular school hours. Then there is always a large supply of material to be obtained free of charge out of doors, for the teacher who knows where to find it. These are only a few of the opportunities that are open to the graded school teacher who has had careful instruction from a trained biologist.

The writer, knowing so well the lack of material for nature-study in the graded schools, has made it one of the requirements for some years past for each student taking courses in the department of biology in this school to prepare an insect collection of one hundred forms classified to the order, and a herbarium of tree leaves of fifty specimens, so that the prospective teacher will not be entirely without material when she reaches her field of labor. The graduates who have returned from time to time to this school have told the writer that they have used this material over and over again in their nature-study teaching in the grades.

In the writer's classes usually there is used some good textbook. Holtz's "Nature-Study" and Hodge's "Nature-Study and Life" have both been found to be very useful. Sometimes simple

informal talks are given to the prospective teachers, but no notes are required, for the writer believes that note-taking by students —for the purpose of preparing a future lesson—lower than college juniors is pedagogically wrong. The emphasis in the work is placed primarily upon the practical side—the laboratory and field work. Living plants and animals are used as much as possible for illustrative material and students are encouraged to plan to do the same in their grade teaching. Detailed descriptive notes in the laboratory have been reduced to a minimum and in place of these the writer has introduced the plan of making many outline drawings with full descriptions of the same which are carefully examined at the close of each laboratory period. This has been done largely for two reasons. The first reason is because the writer has found that his time can be used to far better advantage in behalf of his classes than in correcting piles of notebooks from fifty to sixty students each day. All drawings with accompanying descriptions of the plates are carefully looked over and corrected at the close of each laboratory period. The writer is convinced that the work in connection with the notebook is almost valueless unless carefully checked by the teacher and errors corrected by the students. No written work is done in the writer's classes that is not carefully examined; this he makes an inviolable rule. It is evident that the task of correcting so large a number of papers is no light task; when this is augmented by detailed descriptive notes the problem becomes appalling. It is possible that the writer would require more descriptive written notes, if the state provided more competent assistance for routine work in the laboratory and for the correction of notebooks.[2] The second reason for reducing the amount of written descriptive work in the laboratory is because of its psychological effect on the student. Young students, even those who are already graduates of the high school, frequently become slaves to the notebook. They form the note-taking habit to the great detriment of the real scientific part of their work. The writer has frequently witnessed, both in normal school and college, the loss of mental spontaneity and buoyancy of spirit and an almost entire loss of true interest in nature-study because of the undue importance placed upon the work of the notebook.

The writer wishes to criticise sharply a certain practice that

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[2]The writer has little sympathy for a certain policy that is very common in many schools—the organization of new departments of instruction—when assistance is much needed in those already established.

he has observed by some teachers in laboratory teaching, and that is the correction of notebooks or other work in the laboratory, or in another room close by, while the laboratory work is in progress. This is a glaring pedagogical error. The writer is convinced that there is no form of instruction that requires closer attention on the part of the teacher than that of the laboratory. The instructor should be present throughout the period constantly encouraging, criticising, and helping. The experience of the writer has proven that the laboratory work, if well done, is far more taxing than either the recitation or the lecture. To obtain satisfactory results he has found it necessary to give the laboratory work the closest sort of attention.

During the laboratory periods the writer seldom, any more, uses a manual, but in place of this gives oral directions, having a general plan, or outline of the laboratory work made out before hand. The writer constantly asks questions of his students while the laboratory work is in progress. Frequently the laboratory section is asked to verify the description of a structure, that the teacher has explained, from the specimen in the dissecting pan on the desk before them; or they may be asked to find and describe some structure which the teacher has simply named to them. Not infrequently comparisons are asked for of the same structure observed by a number of different students. Then again several members of the class may be sent to the blackboard to make simple outline drawings of observed structures, in order that comparisons may be made regarding individual observations. The placing of such drawings on the blackboard visualizes difficult points for the benefit of the entire class.

It is absolutely essential that these prospective teachers of nature-study should be instructed as to the manner in which the subject is to be presented in the grades, and also necessary that they should carry away with them a certain amount of biological information; yet it has been the writer's chief concern to arouse and stimulate interest in the wonderful world of nature and to assist them to retain this interest and later to impart it to others.

THE REFRACTION OF STREAM LINES.

By Lindley Pyle,
Assistant Professor of Physics, Washington University, St. Louis, Mo.

The student in physics becomes acquainted with the law of refraction of light at an early stage in his instruction, but the equally interesting law of refraction of stream lines is frequently never brought to his attention. The following simple device permits the investigation of the law of refraction of the lines of electrical current flow at the boundary between two conducting media.

Procure from a tinsmith a piece of *heavily* tinned sheet iron. Cut from it a rectangle, say 25 by 36 cms. Cut out two other rectangles each 25 by 18 cms. Clean the surfaces thoroughly, superpose the smaller rectangles and slip the large one between them. In this way a composite rectangle may be built up, 25 by 36 cms., one half of the sheet being of single thickness, the other half of triple thickness (See Fig. 1). Now place under pressure

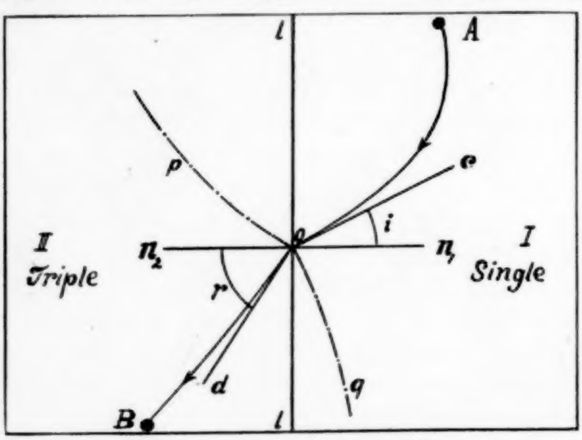

Fig 1

between two flat, hot, iron plates in order to fuse the tinned surfaces together. No solder is to be used, though some soldering paste will prove valuable in bringing about a perfect electrical contact between the superposed layers, particularly at the edge where the triple thickness merges into the single thickness. No trouble will arise if the tinning on the plates be of heaviest quality. Solder binding posts at the points A and B. See Fig. 1.

Send through the device a direct current of approximately five amperes and, in the usual manner, locate an equipotential line crossing the boundary line, *ll*. A pointed conductor connected to one terminal of a moderately sensitive galvanometer is held against the sheet at *p*. A second pointed conductor connected with the other galvanometer terminal is held at such a point *q* that the galvanometer is not deflected. (The contact points must not be warmed by the touch of fingers on account of the resulting generation of thermal electromotive forces.) Maintaining the contact at *p*, locate a number of equipotential points at intervals of half a centimeter and draw a smooth equipotential line through the points. Note the abrupt change of direction at the line dividing the two conducting media.

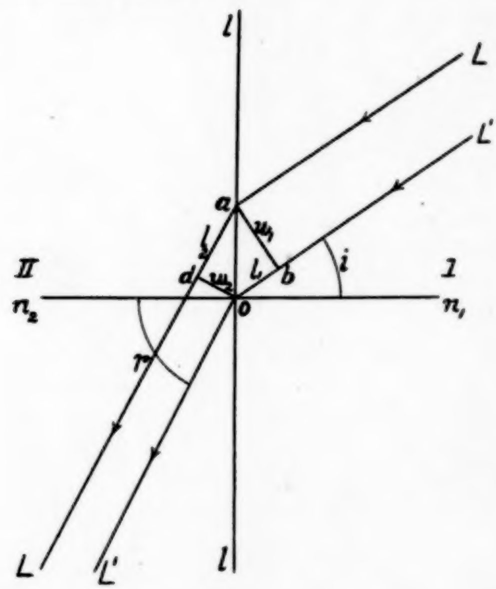

Fig II

Draw *oc* perpendicular to *oq* at the point *o*. Draw *od* perpendicular to *op* at *o*. Then *co* represents the direction of current flow in medium I just before entering medium II, and *od* represents the direction of current flow in medium II just after leaving medium I. Figure I, which is a reproduction on a scale of one to four of the data of an actual experiment, shows a flow line traced from A to B. This flow line was drawn in at right angles to numerous equipotential lines not indicated in the figure.

Draw n_1n_2 normal to ll at o. Let angle $n_1oc = r$. Let angle $n_2od = r$. The figure will show that

$$\frac{\tan\ i}{\tan\ r} = \frac{1}{3}$$

which is the ratio of the electrical conductivity of medium I to that of medium II. This ratio is independent of the location of o along ll.

That this result might have been predicted may be readily shown as follows:

In Fig. II, let LL and L'L' represent two adjacent flow lines in the conducting sheet, the figure being much magnified.

Let C represent the current density at any point in amperes per unit width of the elemental sheet of flow.

Let S represent the resistance of the sheet in ohms per running centimeter for unit width.

ab is an equipotential line in medium I.

od is an equipotential line in medium II.

It thus appears that the fall of potential from b to o is the same as the fall of potential from a to d, or, by Ohm's law,

$$C_1l_1S_1 = C_2l_2S_2 \quad (\mathrm{I})$$

were $l_1 = bo$, and $l_2 = ad$. The subscripts refer to the medium under consideration. The elemental sheet of flow is taken so narrow that the current density is uniform throughout its width.

Further, the quantity of electricity transferred per second across ba is equal to the quantity of electricity transferred per second across od. That is

$$C_1w_1 = C_2w_2 \quad (II)$$

where $w_1 = ba$ and $w_2 = od$.

Dividing equation (I) by equation (II) member by member,

$$S_1\frac{l_1}{w_1} = S_2\frac{l_2}{w_2}$$

$$S_1 \tan i = S_2 \tan r$$

$$\frac{\tan i}{\tan r} = \frac{S_2}{S_1}$$

Since the conductivities, K_1 and K_2, of the two media are inversely as the respective resistances,

$$\frac{S_2}{S_1} = \frac{K_1}{K_2}$$

and $\dfrac{\tan i}{\tan r} = \dfrac{K_1}{K_2}$ as was deduced from experiment.

The refraction phenomena in electric induction, magnetic induction, heat conduction, and other analogous cases, while exhibiting the same law, do not lend themselves so readily to experimental investigation.

PROVISIONAL REPORT OF THE NATIONAL COMMITTEE OF FIFTEEN ON GEOMETRY SYLLABUS.

PRELIMINARY STATEMENT.

At the meeting of the National Education Association in Cleveland in 1908 the mathematics Round Table of the Secondary Department, numbering some two hundred members, unanimously called for a national committee to study and report upon the question of a syllabus for geometry. In December, 1908, the American Federation of Teachers of the Mathematical and Natural Sciences at its meeting in Baltimore authorized the appointment of a national committee of fifteen on geometry syllabus. At the Denver meeting of the National Education Association in 1909, the secondary department authorized the committee which had already been appointed by the American Federation to proceed under the joint auspices of the two national bodies.

The committee thus constituted was made up of eight representatives of secondary schools and seven representatives of universities, as follows: William Betz, East High School, Rochester, N. Y.; Edward L. Brown, North High School, Denver, Colo.; William Fuller, Mechanic Arts High School, Boston, Mass.; Walter W. Hart,[1] Shortridge High School, Indianapolis, Ind.; Frederick E. Newton, Andover Academy, Andover, Mass.; Eugene R. Smith, Polytechnic Preparatory School, Brooklyn, N. Y.; Robert L. Short, Technical High School, Cleveland, O.; Mabel Sykes, Bowen High School, Chicago, Ill.; Charles L. Bouton, Harvard University; Florian Cajori, Colorado College; Herbert E. Hawkes,[2] Yale University; Earle R. Hedrick, University of Missouri; Henry L. Rietz, University of Illinois; David Eugene Smith, Teachers' College, Columbia University; Herbert E. Slaught, *Chairman*, University of Chicago.

After an extended preliminary correspondence in which each member of the committee submitted to all the other members a full statement of his views on all phases of the work in hand, including desirable methods of procedure, the investigations of the committee were conducted in three subdivisions of five members each, the first dealing with "logical considerations," under the chairmanship of David Eugene Smith; the second having charge of "exercises and problems," under the chairmanship of Henry L. Rietz; and the third determining the "lists of theorems," under the chairmanship of Earle R. Hedrick.

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[1]Now Assistant Professor of Education at the University of Wisconsin.
[2]Now Professor of Mathematics at Columbia University.

These subcommittees carried on their work by correspondence and in some cases by meetings of their members during a period of a year and a half, the results being submitted from time to time to the general committee for suggestions and criticisms. A meeting was then held in Cleveland, Ohio, on November 24, 25, 26, 1910, at which were present the three subchairmen, the general chairman, and three other members. All the recommendations of the subcommittees were submitted to a searching examination in the course of which they were either passed, eliminated, amended, or reconstructed till finally substantial agreement was reached on all points; whereupon it was decided to print the report in the form thus evolved and to submit it to one hundred or more selected critics for further suggestions before its final presentation at the San Francisco meeting in July, 1911.

The report contains a historical introduction prepared by Florian Cajori; a section on logical considerations, including axioms, definitions, new terms and symbols, treatment of limits and incommensurables, time and place in the curriculum, purpose of the study of geometry; a section on the grading and distribution of exercises, algebraic and geometric types of exercises, special classes of exercises such as loci problems and problems in concrete setting, and the correlation with other subjects such as arithmetic and trigonometry; a section on types of courses for special classes of students and on preliminary inductive courses in the grades; and finally the syllabus itself exhibiting by means of distinctive forms of type the varying degrees of emphasis which may properly be attached to the different theorems both as to their importance in the logical chain and as to their richness in applications.

In the preparation of this report the various syllabi published in this country and in foreign countries have been consulted. With one of these, the syllabus published by the Association of Mathematics Teachers in New England, this report makes direct comparison, indicating both omissions from, and additions to, that document.

SECTION A. INTRODUCTION.

Attempts Made during the Eighteenth and Nineteenth Centuries to Reform the Teaching of Geometry.

BIBLIOGRAPHY.

The following six books bearing on the history of the teaching

of geometry have been found most useful in making this compilation:

1. V. Bobynin—"Elementare Geometrie," being Chapter XXII in Cantor's *Vorlesungen über Geschichte der Mathematik,* Vol. IV, Leipzig, 1908, pp. 321-402. (Covers the second half of the eighteenth century.) Referred to as "Bobynin."

2. F. Klein—*Elementarmathematik vom höheren Standpunkte aus, Theil II; Geometrie.* Leipzig, 1909, pp. 433-515. Referred to as "Klein."

3. J. Perry—*Discussion on the Teaching of Mathematics.* British Association Meeting, 1901. Referred to as "Perry."

4. H. Schotten—*Inhalt und Methode des planimetrischen Unterrichts.* Leipzig, 1890. Referred to as "Schotten."

5. M. Simon—*Ueber die Entwicklung der Elementar-Geometrie im XIX Jahrhundert.* Leipzig, 1906. Referred to as "Simon."

6. A. W. Stamper—*A History of the Teaching of Elementary Geometry,* New York, 1906. Referred to as "Stamper."

Other useful sources of information on the history of the teaching of geometry are as follows:

1. *Reports of the Association for the Improvement of Geometrical Teaching* (in England). The Association now calls itself "The Mathematical Association," and its present organ is the *Mathematical Gazette.*

2. *Zeitschrift für Mathematischen and Naturwissenschaftlichen Unterricht,* Leipzig and Berlin. Formerly called "Hoffman's Zeitschrift," now "Schotten's Zeitschrift."

3. *L'Enseignement Mathématique. Revue internationale.* Paris.

4. *La Revue de l'Enseignment des Sciences.* Published monthly. Paris.

5. *Nature* (London). See Indexes for "Geometry."

6. D. E. Smith—*The Teaching of Geometry,* Boston, 1911.

7. G. Loria—*Vergangene und Künftige Lehrplane.* Deutsch von H. Wieleitner, Leipzig, 1906.

8. G. Loria—*Della varia fortuna di Euclide.* Rome, 1893.

9. R. Fricke—*"Ueber Reorganisationsbestrebungen des Mathematischen Elementarunterrichts in England."* Jahresbericht d. deutsch. Math. Vereinigung, Vol. XIII, 1904, p. 283, etc.

10. Klein-Schimmack—*Vorträge über den mathematischen Unterricht an den Höheren Schulen.* Leipzig, 1907.

11. *Plan d' études et programmes d' enseignement dans les lycées et collèges de garcons.* Paris, 1903.

12. A. W. Stamper's list of references at the end of his book.

13. The various histories of mathematics.

FRANCE.

France began to maintain a critical attitude toward Euclid as a text-book in geometry for beginners as early as the time of Petrus Ramus (1580). Ramus treated geometry as the art of accurate measurement. In the eighteenth century this spirit of independence was intensified by the publication of Clairaut's *Elémens de Géométrie* (1741), in which surveying and other practical matters received marked attention. In the latter half of the eighteenth century Euclid ceased to be used as a text-book in France.

Williamson, in his edition of Euclid, 1781, criticises Clairaut as follows: "Elements of geometry carefully weeded of every proposition tending to demonstrate another; all lying so handy that you may pick and choose without ceremony. 'This is useful in fortification;' 'you cannot play at billiards without this.' 'You only look through a telescope like a Hottentot until this proposition is read,' with many such powerful strokes of rhetoric to the same purpose. And upon such terms, and with such inducements, who would not be a mathematician? Who would go to work with all that apparatus which I have described as necessary for understanding Euclid, when he has only to take a pleasant walk with Clairaut upon the flowery banks of some delightful river, and there see, with his own eyes, that he must learn to draw a perpendicular before he can tell how broad it is?" About 1836 De Morgan remarks that these arraignments are not "without their force, when directed against experimental geometry as an ultimate course of study, [but] lose their ironical character and become serious earnest, when applied to the same as a preparatory method." De Morgan strongly favors a geometry like Clairaut's as a preparatory course.

The critical attitude of Ramus and Clairaut toward the *Elements* of Euclid brought to the mind of D'Alembert the questions: "What are the elements of a science? What should be the contents of a book called elements?" D'Alembert gives his answers in two articles, "Elémens des sciences," and "Des élémens de géométrie" in the *Encyclopédie méthodique* (about 1784).[3]

[3]*Encyclopédie methodique, Mathématiques I,* 617-625; III, 133-136. We have used the Italian translation of this dictionary, Padova, 1800, and also a full abstract of these articles, given by Bobynin. See Bobynin, p. 325, etc.

D'Alembert distinguishes between two kinds of elements of a science:

(1) If all truths or theorems of a science which are the foundation for all others are brought together, so that these truths or theorems potentially comprise the whole science, then these constitute, when properly coördinated, the *elements* of the science. In geometry, such elements embrace not merely the principles of mensuration and the properties of plane figures, but also the application of algebra to geometry, and the differential and integral calculus in its application to curved lines.

(2) The elements of a science may be defined also as comprising those truths or theorems which treat the subject matter in the simplest way, and which constitute, together with their deductions, a detailed study of the simplest parts of the science. By the elements of geometry, elements of this kind are usually meant; they include only the properties of plane figures and the circle.

Dissatisfied with the elements of geometry known in his day, D'Alembert sets up the following demands which such texts should fulfill:

(1) The text should develop the subject along the path pursued by the discoverers of the science, so as to show the truths in their natural relations to each other.

(2) The usual division of the subject into longimetry, planimetry, and stereometry does not provide for the circle and sphere and is therefore inadequate. The division into plane geometry and solid geometry, D'Alembert does not consider at all. He suggests the division into the geometry of the straight lines (considered with respect to position and relative magnitude) and circles, the geometry of surfaces and the geometry of solids. The straight line and circle must be taken up together. The circle renders immense service in considering the position of lines. The measurement of angles by circular arcs and the principle of congruence constitute the basis of the first part of the geometry of lines, upon which other theorems of this part rest. The second part in the geometry of the straight line has as its fundamental theorem the one on the section into proportional parts of two sides of a triangle by a line parallel to the third side. This involves incommensurables.

(3) Incommensurable relations must be treated by the apagogic method, according to which it is shown that one ratio cannot be greater or smaller than a certain other ratio, hence it

must be equal to that other ratio. He uses this for the following reasons: Incommensurable magnitudes involve the idea of the infinite and therefore, he claims, cannot be treated by any direct method. Notwithstanding this difficulty presented by incommensurable lines, he maintains that they should be taken up early in geometry, because of their importance. He states that the whole theory of incommensurables demands only one theorem, concerning the limits of quantities, viz: "Magnitudes which are the limits of one and the same magnitude, or magnitudes which have one and the same limit, are equal to each other." In the geometry of the circle, of surfaces and solids, he feels that the method of exhaustion or that of limits should be used.

(4) A suitable text on the elements of geometry can be prepared only by a mathematician of the first rank. D'Alembert complains that most elementary geometries are written by men of little ability.

(5) To lay down definitions at the beginning without any analysis of the subject is not only contrary to sound philosophy but contrary to the natural march of thought.[4] Axioms are useless.

Ideas similar to those of D'Alembert are embodied in a text on geometry by Louis Bertrand of Genève,[5] who in Berlin had been close to Euler. Bertrand's book antedated D'Alembert's articles in the Encyclopédie Méthodique. Like D'Alembert he divides geometry into three parts: (1) Geometry of line and circle, (2) Measurement of parts of a plane bounded by straight lines and circles, (3) Measurement of curved surfaces and solids. Bertrand ignored the classification of geometry into plane and solid. His second theorem is: "When two planes intersect, their common section is a right line." The straight line and circle are taken up together at the beginning as D'Alembert would have it. The incommensurable case is treated by the *reductio ad absurdum* method. In the latter part of the geometry he uses also the method of exhaustion. Bertrand reduces the number of theorems, in one instance, by replacing theorems on the mensuration of prisms, pyramids, cylinders, cones, and spheres by the corresponding problems.

Bertrand's work was published in two unwieldy volumes and had little sale, yet exercised some influence, particularly upon Lacroix, whose *Cours de Mathématiques,* published at the close

[4]See "Axiome" and "Courbe" in *Encycl. Méth.*
[5]*Développment nouveau de la partie élémentaire des mathématiques*, Genève, 1778.

of the eighteenth century, has been used until recently. Lacroix divides his geometry into geometry of the plane and geometry of space, and does not follow D'Alembert closely. According to Lacroix there are only two kinds of theorems that should find a place in an elementary geometry: (1) Theorems necessary for the comprehension of the line of argument, developed synthetically. (2) Theorems which grow out of the practical operations in geometry (drawing and measuring). He objects to placing all axioms at the beginning, believes in the omission of the definition of an angle, favors "a straight line is the shortest path between two points" as growing out of the child's experience, and uses the apagogic method for incommensurables.

Another author of note was Bézout, who followed D'Alembert's plan quite closely, but was criticised for his lack of rigor and for his endeavor to lighten the work of the examiner as well as of those being examined.[6]

The most celebrated work on elementary geometry is that of Legendre (1794). He came nearest to fulfilling D'Alembert's requirement that the elements be written by a mathematician of the first rank. He does not follow D'Alembert's plan for a book on geometry, nor does he heed the philosophic demand that the author should follow the path of the originators of the science. Impressed by the lack of rigor in the works of his day, he aims at greater rigor and approaches closer to Euclid than his predecessors had been. He does not divide geometry in the manner of D'Alembert and Bertrand. Like Euclid, Legendre begins with definitions and axioms. The first four chapters are given to plane geometry, the last four to solid. The first book treats of the equality of angles and triangles, the second of the circle and the measurement of angles, the third of proportional figures, the fourth of regular polygons and the measurement of the circle. Legendre uses in measurement the terms *equal* and *equivalent*. He uses the *reductio ad absurdum* method for incommensurables and the method of exhaustion for curved lines.

What was it that made this book so successful? In the first place must be mentioned his great clearness of exposition and his attractive style. A great advance of Legendre over Euclid was the fuller treatment of solid geometry. He leans less toward logic and more toward intuition than does Euclid. In place of Euclid's famous fifth book on incommensurables, Legendre borrows rational and irrational numbers from arithmetic, even

[6]Bobynin, p. 355.

though in arithmetics no scientific treatment of those subjects was given in his day. A theorem true for rationals is assumed to be true for irrationals. Thus, if A : B = C : D, then AD=BC in all cases. Klein says that this is in accordance with the practice of the best mathematicians of his day, that even Lagrange works out the expansion of $(x+h)^n$ when n is rational and assumes the results thus obtained to be true for irrational values of n. Legendre stands for a fusion of geometry, not only with arithmetic, but also with trigonometry. As late as 1845 Legendre's geometry still contained trigonometry, but as Klein remarks,[7] the trigonometry and the practical applications of geometry were gradually filtered out. Comparing A. Blanchet's edition of 1876 with an edition of 1817, we find also that the twelve "notes" on topics of elementary geometry, covering 55 pages in the older edition, are omitted in the later edition. The later edition has a somewhat fuller treatment of solid geometry and a list of exercises in original proofs, loci, and constructions. Other notable changes were made in the 1845 edition by J. B. Balleroy and A. L. Marchand. They state that Legendre uses the *reductio ad absurdum* method to excess, a method which "convinces but does not satisfy the mind." Legendre's text is, however, left intact, alternative proofs being given in notes at the end. These alternative proofs, as well as the proofs given in the modified text of the 1876 edition, are rough applications of the theory of limits.

During the first half of the nineteenth century, and even later, the works of Legendre, Lacroix, and Bézout were used extensively in France. In later editions less stress was laid upon practical applications and numerical computation. Otherwise few changes occurred. In general, school organization, based on the regulations of the time of Napoleon I, was quite fixed in France until 1870. France has a rigid centralization of authority in education. If the "Conseil d' instruction supérieure" decides upon a change, the whole country adopts it at once. As compared with the German, the French teacher has little individual freedom. France is a country with a "system of revolutions from above."[8] Since 1870 the movement has been toward greater individual freedom. The later tendencies in geometry are imaged in the work of Rouché and de Comberousse, which contains a large amount of new material and meets the demands of the one

[7]Klein, p. 470.
[8]Klein, p. 457.

year course of the *classe de mathématiques spéciales* during which as much as seventeen hours per week are given to mathematics and a degree of specialization is allowed in preparation for university courses, as in no other country. In 1902 and 1905 new official courses of study were adopted in France in which greater stress is laid upon graphic representation, the idea of a variable and a function, and upon the practical applications of mathematics. The introduction of the derivative in algebra in the regular secondary classes, the use of motion in geometry, and emphasis on geometric drawing are other characteristic features of this reform. This new tendency is mirrored in the geometry of E. Borel, a remarkable book, in which the practical receives due emphasis and in which intuition meets with fuller recognition. With Borel the concept of motion is prominently used. There is an introduction of eight pages on the use of the ruler, compasses, and protractor, and ten pages on the mensuration of surfaces and solids, treated empirically. Applications are skillfully interwoven with theory, throughout the book. He has well-selected practical exercises involving symmetry, the nets of regular polygons, the use of pulleys, and so on. Algebraic geometry and the development of metric properties come last in the book. He introduces the rudiments of trigonometry. The usual division into plane geometry and solid geometry is not rigidly maintained. A similar tendency is shown in the recent geometries by C. Bourlet (1908), Fort and Dreyfus (1908), and Niewenglowski (1910).

A parallel and somewhat different tendency in France is seen in the geometry of Ch. Méray of Dijon, which was first brought out in 1874 but has only in recent years received much attention. Méray represents the severely logical mode of exposition;[9] he uses in his proofs no fact of observation which has not been previously set down in an axiom; he formulates a complete list of axioms, but introduces each only when it is needed; nor does he aim to limit their number to a minimum. Characteristic of Méray is the complete fusion of plane and solid geometry, and the use of motion, not only as a means of proof, but also to define parallels. Recently there has been considerable discussion in France on the question whether in laying the foundations to geometry, *motion* should be used or not. The defenders of a static theory of parallels claim that motion cannot be visualized on the board, rendering intuition more difficult. The defenders of the kine-

[9]Klein, p. 475.

matic theory advocate the use of movable figures.[10] Prepared under the influence of Méray, so far as the use of motion is concerned, are the geometries by Borel, Bourlet, Fort and Dreyfus, and Niewenglowski, above mentioned.

Influenced by the Perry movement in England and America, France is experimenting on the laboratory method of instruction.[11] A mathematical laboratory has been recently founded by J. Tannery and E. Borel at the Ecole Normale supérieure in Paris in order to give prospective instructors an insight into the possibilities of this method of teaching.

GERMANY.

Klein[12] expresses surprise that, during the Renaissance, Euclid should have come to be looked upon as a text suitable for the first introduction in geometry. Perhaps the reason for this attitude toward Euclid lies in the fact that geometry was first taken up in the universities by students of maturer years. As geometry came gradually to be taught to younger and younger pupils, Euclid was still retained. Thus the misconception arose that Euclid was a suitable geometrical text for young boys.

While D'Alembert formulated his ideas on elementary goemetry in France, A. G. Kästner evolved in Germany a type of his own, in his work, *Anfangsgründe der Arithmetik, Geometrie, Trigonometrie und Perspectiv,* Göttingen, 1758. Kästner begins with definitions and axioms in Euclidean style, develops the geometry of the plane (69 pages) and ends this part with practical applications (47 pages). The second part of the geometry begins with the geometry of space (60 pages), continues with 31 pages given to plane trigonometry and its applications to the solution of triangles, and with 9 pages of practical geometry. Then follow spherical trigonometry and 24 pages on perspective. The method of exhaustion is used. It was the opinion of Kästner that "the newer works on geometry lose the more in clearness and thoroughness, the farther they depart from Euclid." He complains that modern authors, particularly the French, have departed from the ancient rigor, "to make the study of mathematics easier for people whose main occupation is not study, namely for soldiers."

Not without interest is W. J. G. Karsten's *Lehrbegriff der*

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[10] Schotten, *Zeitschrift*, Vol. 40, 1909, p. 445; *La Revue de l'Enseignement des Sciences* many articles in Vols. 1-3.

[11] Schotten, *Zeitschrift*, Vol. 40, 1909, p. 444-5; *L'Enseignement Mathématique*, 11, p. 206.

[12] Klein, p. 434, 435.

gesamten Mathematik, in eight volumes, 1767-77, the first two volumes of which are given to arithmetic and geometry. Karsten begins with arithmetic, then proceeds to plane geometry, closing with simple arithmetical applications. He proceeds thereupon to solid geometry, returns to arithmetic, and gives the rudiments of algebra with logarithms, followed by trigonometry and its applications to plane geometry. Finally are given the rudiments of spherical trigonometry and a fuller treatment of solids. Nowhere are heavy demands made upon the pupil. That this exposition was intended for students of university grade rather than those in the preparatory school, testifies to the low state of mathematical instruction in German universities of the eighteenth century. Close relation between arithmetic, geometry, and trigonometry is also maintained in the works of J. G. Büsch (1776) and G. S. Klügel (1798), the aim being to make the subject easy of comprehension.

In the nineteenth century, until near its end, advanced mathematicians in Germany took little or no part in the improvement of the teaching of elementary mathematics. In geometry, Euclid's text was not usually taught, but the dogmatic method of Euclid was in vogue during the first half. About the middle of the century Euclid's order of the theorems came to be criticised as chaotic. It is interesting to see the Germans attack Euclid's order as arbitrary and the English defend it as the only order worthy of serious consideration. The grouping of theorems according to subjects came to be discussed in Germany.[13] The advocacy of object teaching by Pestalozzi, the championing of Pestalozzianism by Herbart, the attacks upon mathematical reasoning and particularly upon Euclid that were made by Schopenhauer[14] conspired to influence the teaching of geometry.

About 1860 the genetic method (called "heuristic" when the inventional side was emphasized) came to be discussed, which makes a plea of being a natural method, since it incites self-activity in the pupil. With the genesis of a theorem the pupil sees intuitively its inner relation to other theorems; he not only sees whence he came but also whither he is going; the reader of Euclid is blindfolded, so to speak, and then somehow transported to the next station. It is difficult to prepare text-books for the genetic method. The teacher by careful questioning one moment leads the student, the next moment follows him, and no one can

[13] Schotten, p. 11.
[14] Klein, p. 503.

foresee the exact path which this mode of advance will mark out. It is not strange, therefore, if many teachers proceeded heuristically while the texts retained mostly the dogmatic form.[15] Moreover, experience made it plain to teachers that the dogmatic statement of theorems has a high mnemotechnic value.[16] While the genetic method in its pure form has not succeeded in establishing itself, it has exerted a strong influence by shifting the emphasis from the memorizing of proofs to the cultivation of originality and logical reasoning.

Another movement that sprang from the teachings of Pestalozzi and Herbart was the adoption of preliminary courses on observational geometry and drawing, about 1870. Such courses had been recommended long before this time. This movement was stronger in Germany than in England and France. In their propædeutic courses the geometry of solids was to receive consideration and a taste of the genetic method was recommended. The pupils acquired dexterity in the use of ruler and compasses. Propædeutic courses have maintained their place to the present time.

Herbart made strong endeavors to remove the superstition that had arisen in early days when Euclid was placed in the hands of young and immature students, to the effect that mathematics could be learned only by a few pupils endowed with special gifts. According to his view the fault lies as a rule in the abstract character of the early instruction; the introduction of propædeutic courses and the greater emphasis upon "Anschauung" at all stages had shown that most students can master mathematics. Whether "amathematicians" do exist in rare instances, is a question which Klein refers to experimental psychologists for reply.[17]

A third movement agitated in Germany, was in favor of the introduction into elementary instruction of the concepts of the modern projective geometry. It originated about 1870.[18] The criticism was made that Steiner, Möbius, and von Staudt had been so busy with their researches as to make no attempt to reform elementary instruction, and that text-book writers had ignored the researches of these great men. The leaders in this attempt to incorporate modern methods were Schlegel and Fiedler. A concomitant of this programme was the breaking down of the division of geometry into plane and solid, and the effort by the use

[15] Schotten, p. 96.
[16] Schotten, p. 13.
[17] Klein, p. 499.
[18] Schotten, p. 18.

of models, etc., to make geometry more concrete. To effect this reform, a number of texts by Schlegel, Müller, Kruse, Becker, Worpitzky, Henrici, and Treutlein sprang into existence.[19] Aside from the production of interesting text-books this agitation has had little success. The books in question were seldom used.[20] Can it be that D'Alembert's dogma is, after all, based upon truth —the dogma that the historical order of development of geometry is the pedagogical order; that is, the easiest approach to the science for the young mind? Are the concepts of projective geometry more difficult to grasp than those of the older geometry, or did the texts just named overtax the pupils, and perhaps in other ways violate the demands of sound pedagogy?

Most interesting are the statistics gathered in Prussia in 1880 which showed the following distribution of geometrical texts: Kambly was used in 217 institutions; Koppe in 54; Mehler in 44; Reidt in 29, while 55 texts were used in one institution each. Kambly's "clever but unscientific book" was first issued in Breslau in 1850 and a few years ago reached the 101st edition in the revision by Roeder. Koppe was looked upon as an inferior work, yet it enjoyed great popularity. On the other hand, books like those of H. Müller and even Henrici and Treutlein seldom passed beyond the second edition. This most astonishing success of works considered as scientifically inferior, requires explanation. Schlegel says[21] "that the quality of the books most widely adopted allows one to draw an inference respecting the scientific level of the instruction generally reached in that subject." But this remark considers merely one phase of this question. May not the mass of teachers have had a feeling or insight concerning text-books which involved questions of intuition or other psychologic matters that the writers of the more scientific books overlooked? Simon[22] points out that until recently the German teacher, unlike the French, enjoyed complete freedom in teaching, and that small texts, like Kambly, allow his individuality much wider play.

Kambly's "Elementar-Mathematik" was made up of four parts: first, arithmetic and algebra; second, planimetry; third, plane and spherical trigonometry; fourth, stereometry. Of interest here, is the interpolation of trigonometry. We have before us Kambly's *Planimetrie*, 43d edition, Breslau, 1876. Among the points of popularity we mention the following:

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[19]Schotten, p. 19.
[20]Schotten, p. 20.
[21]Schotten, p. 21.
[22]Simon, p. 25.

1. The book contains only as much matter as a class can conveniently finish in one year. Skipping parts of a book, says Kambly, has a bad effect upon both pupils and parents.

2. The diction is clear and simple. Mathematical symbols are used freely. The setting of the type is such as to enable the eye more quickly to see the relations set forth.

3. The arrangement of the book is such as to allow the teacher much freedom. He may, for instance, omit incommensurables altogether, or else substitute for certain proofs in the regular text others given in the footnotes where rough proofs are found for incommensurable cases.

4. Easy arithmetical applications, original theorems, and original constructions are given at the end of the book, so that some, or all, may be conveniently taken or omitted, according to the preference of the teacher.

Koppe's *Planimetrie* made somewhat greater demands upon the powers of the pupil than did Kambly, but incommensurables were treated only in footnotes or in remarks following the proofs of theroems. In Lübsen's *Elementar-Geometrie,* I have not been able to find a reference to incommensurables. It differs from Kambly and Koppe in having better figures and in having them on the page where they are needed, instead of the end of the book on separate sheets that unfold out. A clever feature in Lübsen are the practical applications introduced from the very beginning. How to run a straight line over undulating country by the use of poles, is explained in several diagrams on the first pages. Other figures show how to determine the distance between points on opposite banks of a river.

Since about 1890 the activity of Felix Klein of Göttingen, in mathematical reform, has been very great. For the first time since the death of Kästner, is the influence of university professors upon the teaching of elementary mathematics in Germany beginning to be strongly felt. Among the defects of geometrical instruction, he points out the insufficient fusion of the various branches of elementary mathematics.[23] Thus, too little attention is given to drawing of solids and to projection, to the idea of motion in a figure to replace Euclidean rigidity, to the fusion of arithmetic and geometry, to the introduction of the coördinate representation of analytics. On the other hand, the construction of triangles from given data, is over emphasized,[24] as is also the study of the curious

[23]Klein, p. 439.
[24]Klein, p. 442.

points and lines in the geometry of the triangle. This last criticism applies even more strongly to English text-books.

Klein points out that modern demands in geometric teaching, *first*, emphasize the psychologic point of view,[25] which considers not only the subject matter, but also the pupil, and insists upon a very concrete presentation in the first stages of instruction, followed by a gradual introduction of the logical element; *second*, call for a better selection of the material from the viewpoint of instruction as a whole; *third*, insist on a closer alignment with practical applications; *fourth*, encourage the fusion of plane and solid geometry, and of arithmetic and geometry.[26] The *Lehrbuch der Mathematik nach modernen Grundsätzen* by Beherendsen and Götting, a secondary school text published in 1909, carries out Klein's ideas on the teaching of algebra and geometry.

A piece of research of vital importance in the advanced study of geometry is the *Foundations of Geometry*, brought out in 1899 by Professor Hilbert of Göttingen.[27] Though widely read by mathematicians, it has exerted no direct influence upon elementary teaching in Germany. It has been felt that this mode of treatment is not suitable for pupils first entering upon demonstrative geometry.

ITALY.

Since the unification of Italy, great mathematical activity has existed in that country. Before that event, very different practices in geometrical teaching existed in different parts of the country.[28] In 1868 Cremona and Battaglini were members of a government commission to inquire into the state of geometrical teaching. They found it unsatisfactory, and the number of bad text-books so great, and so much on the increase, that they recommended for classical schools the adoption of Euclid, an edition of which was brought out by Betti and Brioschi. Later other works of scientific merit replaced Euclid. Cremona's great emphasis upon projective geometry reached from the universities down into secondary schools. A typical work is that of A. Sanna and E. d'Ovidio, 1869, which uses the theory of limits and retains the division of geometry into plane and solid. It stands closer to Euclid than to Legendre. The blending of

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25 Klein, p. 435.
26 Klein, p. 437.
27 D. Hilbert, "Grundlagen der Geometrie" in *Festschrift zur Feier der Enthüllung des Gauss-Weber-Denkmals in Göttingen*, Leipzig, 1899.
28 Simon, p. 43.

plane and solid geometry, which received great emphasis in Italy, is typified in the *Elementi di Geometria* of R. de Paolis, 1884.

A very remarkable school came into being in Italy, the purpose of which is to render geometry still more rigorous than in the Euclidean text. Starting with a single basic concept, the point, all other concepts are to be logically developed. This movement is typified in the works of G. Veronese.[29] Of elementary works he has prepared *Nozioni Elementari di Geometria Intuitiva,* 1902, and *Elementi di Geometria,* 1904, the first of these being a propædeutic work. Demonstrative geometry is taken up in Italy with older pupils than in Germany and the United States; hence works of greater rigor can be used. Veronese endeavors to state all the necessary postulates of geometry, no matter how obvious, as for instance, "There exist *different* points," to make it plain that we do not consider a geometry in which only one point exists.[30] As regards the selection of material, Veronese confines himself mainly to that of Euclid, thus receding from the tendency of the School of Cremona. He avoids all fusion with arithmetic. Somewhat similar in character is the *Elementi di Geometria* of F. Enriques and U. Amaldi, 1905.

The effort at rigor, due to Veronese, has been intensified in the great school of Peano, which endeavors to eliminate all intuition. It seems that this school has influenced even elementary instruction and the teaching in technical schools.[31] This recent Italian emphasis upon extreme rigor has led to deplorable results with the less gifted pupils, and a reaction appears to be setting in. Under the leadership of Loria and Vailati a movement is on foot favoring greater emphasis upon intuition, the introduction of some modern geometrical notions, the fusion of geometry with arithmetic, and the concession to the demands for practical applications made by this age of industrial development. In fact, Italy is entering upon a reform much like that of Germany and France.[32]

ENGLAND.

Roger Bacon says that toward the close of the thirteenth century the definitions and a few of the theorems in geometry were studied by some pupils at Oxford.[33] About 1570 Sir Henry

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[29] Klein, p. 482.
[30] Klein, p. 483.
[31] Klein, p. 486.
[32] For additional details see W. Lietzmann's article in Schotten's *Zeitschrift,* Vol. 39, p. 177-191; Vol. 40, p. 227-228.
[33] Ball, *Mathematics at Cambridge,* 1889, p. 3.

Savile began to lecture at Oxford on Greek geometry, and in 1619 Briggs at Cambridge on Euclid. In 1665, Isaac Barrow at Cambridge prepared a complete edition of Euclid, which was the standard for fifty years. Gow says that "the seventy years or so, from 1660 to 1730, when Wallis and Halley were professors at Oxford, Barrow and Newton at Cambridge, were the period during which the study of Greek geometry was at its height in England." [34] In 1703, William Whiston became the successor of Newton at Cambridge. He brought out an edition of Tacquet's Euclid. Robert Simson's edition of Euclid first appeared in 1756. Simson was professor of mathematics at the University of Glasgow. In the universities of Great Britain, Euclid met with no competition. Ward's *Young Mathematician's Guide,* 1707, may have been used to some extent, but probably more for its arithmetic and algebra than for its geometry. Practical men, holding positions as excise officers, had to be familiar with practical geometry. For them practical treatises existed, some of which gave explanations of the slide rule. A departure from Euclidean rigor might be expected in the education of men for the army or navy. We have seen that Kästner criticised the French for making mathematics easy for men interested in war. England has had since 1722 an academy at Portsmouth where men spent one or two years studying navigation, drawing, etc. England has had also, since 1741, a military academy at Woolwich, where sons of noblemen and military officers were taught fortification, gunnery, and mathematics. Among the mathematical professors at Woolwich, during the eighteenth century, were Thomas Simpson, John Bonnycastle, and Charles Hutton, all three authors of textbooks, including geometries. Hutton's works went through several editions in the first half of the nineteenth century. From this it is evident that Euclid did not hold universal sway in England. Yet the forces opposing him were utterly unable to dislodge him.

In 1822, Sir David Brewster brought out an English translation of Legendre's geometry. Did teachers rally in favor of the introduction of this text? We shall see that DeMorgan suggested the use of some parts of it on solid geometry; DeMorgan deplored that solid geometry was seldom or never taught before trigonometry. But otherwise we are not able to find any serious reference to this translation of Legendre.

During the second half of the eighteenth century England had come to be the only country where Euclid was practically the only

[34] Gow, *History of Greek Geometry*, Cambridge, 1884, p. 208.

geometrical text used. During the eighteenth century the average age of freshmen in the English universities was gradually increasing, and perhaps at this time, Euclid passed from the universities to the lower schools. There is no explicit proof, however, that in the great "Public Schools" Euclid was studied before the nineteenth century.[35]

Very recently[36] some interesting information has been published about one of the "public schools"—Christ Hospital—which paid more than usual attention to mathematics in the courses for boys preparing to enter the royal navy. It seems that as early as 1680 such boys were required to study the earliest parts of the first book of Euclid, the 10th, 11th, and 12th propositions of the sixth book, and to learn arithmetic. Perhaps this represented all the theoretical mathematics taught, for Sir Isaac Newton, whose advice about changes in the course was sought, notes the following omissions: There was no "symbolic arithmetic," no "taking of heights and distances and measuring of planes and solids," no "spherical trigonometry," nothing of "Mercator's chart." In other "public schools" probably no courses in geometry were given during the eighteenth century. Says Stamper: "It was not until about the middle of the nineteenth century that the study of Euclid became common in the secondary schools of England."

It would be instructive to secure more information explaining how it was possible for Euclid to maintain its supremacy as a text, when geometry was being transferred from the universities to the schools. What were the experiences of teachers in secondary schools with the Euclidean text? The desirability of modifying Euclid must have arisen early, for in 1795 John Playfair brought out a revised Euclid containing the first six books and adding the computation of π and a book on solid geometry drawn from modern sources. Playfair endeavored to give the geometry a form which would render it more useful. Euclid's fifth book, which had never been used successfully with beginners in geometry, as far as we can ascertain, was modified by Playfair by replacing Euclid's prolix explanations by the more concise language of algebra. But Playfair did not try to change the nature of the reasoning. Had there been a strong movement against Euclid in England at this time, Playfair would probably have joined it. In his review of Leslie's *Geometry* in the Edinburgh Review, Vol. 20, 1812, p. 79, he says: "A question has been

[35] Stamper, p. 88.
[36] "A School Course in Mathematics in the XVII Century" by W. W. R. Ball in the *Mathematical Gazette*, Vol. V, 1910, Part I, pp. 202-205.

sometimes agitated whether it is most advantageous, for the study of geometry, to possess a number of elementary treatises, or to have one standard work, like that of Euclid . . . the same lessons are not suited to every intellect, and on these accounts it may be of advantage that different elementary texts should exist. We are very much inclined to the latter opinion."

William George Spencer's unique booklet on *Inventional Geometry* was brought out about 1830 or 1835, but "received but little notice" at that time. A noteworthy device for aiding the young mind through sensuous stimulus was the use of colored diagrams, suggested by Oliver Byrne, in his edition of Euclid, London, 1847. The failure of this book is doubtless due to the want of moderation in the use of colors.

The ablest writer on the teaching of elementary geometry during the first half of the nineteenth century in England, was Augustus DeMorgan. His articles published in the *Quarterly Journal of Education,* in 1831, 1832, and 1833, display a pedagogical insight which would have prevented many calamities in English teaching, had his views been more promptly and widely accepted. Elsewhere we quoted DeMorgan's remarks on Williamson's criticism of Clairaut's geometry, which showed that DeMorgan firmly believed in a preliminary course in geometry, as an introduction to a logical course like that of Euclid. It will appear that England was the last country actually to introduce propædeutic courses in elementary instruction.

DeMorgan did not hesitate to recommend radical changes in Euclid. Here is what he said in 1831, in an article in the *Quarterly Journal of Education,* entitled "On Mathematical Instruction:"

"With regard to the fifth book of the Elements, we recommend the teacher to substitute for it the common arithmetical notions of proportion. Admitting that this is not so exact as the method of Euclid, still, a less rigorous but intelligible process is better than a perfect method which cannot be understood by the great majority of learners. The sixth book would thus become perfectly intelligible."

Two years later, in an article in the same journal "On the Methods of Teaching the Elements of Geometry," DeMorgan dares to suggest that certain parts of Legendre might be profitably substituted for parts of Euclid. "The eleventh book of Euclid may, in our opinion, be abandoned with advantage in favour of more modern works on solid geometry, particularly that of

Legendre, which the English reader will find in Sir David Brewster's Translation." In the same article DeMorgan gives utterance to a difficulty experienced by young students, which has been referred to by many writers in different countries, the *reductio ad absurdum.* DeMorgan says: "The most serious embarrassment in the purely reasoning part is the *reductio ad absurdum,* or indirect demonstration. This form of argument is generally the last to be clearly understood, though it occurs almost on the threshold of the elements. We may find the key to the difficulty in the confined ideas which prevail on the modes of speech there employed." As regards the difficult fifth book, DeMorgan said, in 1833, "We would say to all, teach the fifth book, *if you can;* but we would have all remember that there is an *if.*" In another place he adds: "We strongly suspect that Euclid, as studied, does as much harm as good." To the credit of teachers be it said, that the fifth book was quite generally omitted. But DeMorgan's activity in this line did not end here. In 1836 he published *The Connexion of Number and Magnitude; An attempt to explain the fifth book of Euclid.* For fifty years this tract was not duly appreciated; later it began to wield a wide influence; it is on this tract that the substitute for the fifth book given in the Syllabus of the Association for the Improvement of Geometrical Teaching is modeled; it is on this tract that the revised fifth book in the more recent editions of Euclid by Nixon and by Hall and Stevens is based.

The need of modifying the text of Euclid is brought out by DeMorgan in the *Companion to the British Almanac* of 1849, page 20, as follows: "If the study of Euclid has been almost abandoned on the continent, and has declined in England, it is because his more ardent admirers have insisted on regarding the accidents of his position as laws of the science."

How little influence DeMorgan's views wielded in England before about 1870 as regards the revision of Euclid's fifth book and the study of solid geometry, appears from the fact that the most popular edition of Euclid for many years, was the one brought out in 1862 by Todhunter. This author reproduces Simson's text, though he greatly assists the pupil in overcoming the difficulties by breaking up the demonstrations into their constituent parts. In an Appendix are given notes, supplementary propositions and original exercises. Todhunter was quite out of sym-

pathy with the purposes of the Association for the Improvement of Geometrical Teaching.[37]

Opponents of Euclid existed in England at all times. Thus in 1860 W. D. Cooley brought out a rival text. Eleven years later he expressed himself regarding this venture as follows:[38]

"In 1860 there was published for me, by Messrs. Williams and Norgate, a little volume entitled, *The Elements of Geometry Simplified and Explained,* adapted to the system of empirical proof, and of exhibiting the truth of theorems by means of figures cut in paper. It contains in 35 theorems the quintessence of Euclid's first six books, together with a supplement not in Euclid. There was no gap in the sequence or chain of reasoning, yet the 32nd and 47th propositions of Euclid were, respectively, the 3d and 17th of my series. This book proved a failure, for which several reasons might be given, but it will be sufficient here to state but one, namely, that it came forth ten years before its time."

The reformers found a champion in Sylvester, who in 1869, before Section A of the British Association exclaimed: "I should rejoice to see mathematics taught with that life and animation which the presence and example of her young and buoyant sister (natural science) could not fail to impart, short roads preferred to long ones, Euclid honorably shelved, or buried 'deeper than e'er plummet sounded' out of the schoolboy's reach. . . . ." The reform forces finally organized themselves, in 1871, into the "Association for the Improvement of Geometrical Teaching" (A. I. G. T.).

It is a curious circumstance that England's great mathematician, Arthur Cayley, opposed this reform movement. His admiration for Euclid was so ardent that he even expressed a preference for the original treatise without Simson's additions. In the opinion of Langley, Cayley "overshot the mark and his opposition told in favor of the Association."[39]

The second *Report* of the A. I. G. T. recommended practical exercises in geometrical construction, easy originals and numerical examples. Two years were given to the preparation of the Geometrical Syllabus on proportion. A double syllabus was prepared: A Syllabus on Geometric Constructions, and a Syllabus on Plane Geometry. Most of DeMorgan's suggestions[40] on the revision of the fifth book of Euclid were adopted.

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[37]See *Conflict of Studies*, by Todhunter, London, 1873.
[38]Nature, Vol. 4, 1871, p. 486.
[39]*Fifth Report of the A. I. G. T.*, p. 21.
[40]*Companion to the British Almanac*, 1849, pp. 5-20.

This Society, after long labors, finally issued a substitute text, *The Elements of Plane Geometry.* This was not used at home, but was used with success in the British colonies. Klein expresses himself, as follows, in regard to it: "This is essentially merely a smoothed down and polished presentation of the first six books of Euclid's elements; thus the rough places at the beginning of the first book . . are removed by a consistent use of the concept of motion, but in general the sequence and the contents of Euclid are adhered to, in deference to the examinations. It is therefore only a tame reform, that is here attempted; nevertheless, it has met with sharp opposition by the adherents of the old English system. As proof of this, I refer to an amusingly written book of Dodgson, *Euclid and His Modern Rivals.*" Here Euclid comes out victorious, and all reformers, particularly Legendre and members of the A. I. G. T., are put to the rout. J. M. Wilson's *Elementary Geometry,* 1st edition, 1869, came in for a large share of the criticism. At Oxford, where Dodgson had given instruction in geometry for many years, this same Wilson had, at one time, read a critical paper before the Mathematical Society, on "Euclid as a Text-Book of Elementary Geometry." Wilson was a prime mover in the organization of the A. I. G. T.

Since about 1870, many editions of Euclid have been printed containing revisions with the object of better adapting Euclid to school use. They exhibit all possible gradations of departure from the original text. There appeared sequels to Euclid like that of F. Casey. Professor Klein expresses himself in regard to these as follows: "The necessity has been felt to consider modern research, going beyond Euclid; this has been done by pressing it by force into the rigid Euclidean form, whereby a good part of the modern spirit is, of course, lost."[41]

During thirty years the A. I. G. T. appeared to have accomplished comparatively little. It had secured the concession that proofs different from Euclid's shall be accepted in examinations and had brought about a sentiment favoring some modification and enrichment of the Euclidean text. In reality, it had accomplished much more, for it had prepared the way for the great agitation of 1901, known as the Perry Movement, which called for a complete divorce from Euclid. The discussion of the teaching of mathematics at the Glasgow meeting of the British Asso-

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[41]Klein, p. 447.

ciation marks an epoch. The following are suggestions and criticisms that were contained in Perry's Syllabus.[42]

1. Experimental geometry and practical mensuration to precede demonstrative geometry. Use of squared paper. Rough guessing at lengths and weights to be encouraged.

2. Some deductive reasoning to accompany experimental geometry.

3. More emphasis on solid geometry; this subject has been postponed too long.

4. Adoption of coördinate representation in space.

5. The introduction of trigonometric functions in the study of geometry.

6. Emphasis upon the utilitarian parts of the subject.

7. Examinations conducted by any other examiner than the pupil's teacher are imperfect examinations.

In criticism of previous practices, Perry held that a boy should be educated through the experiences he already possesses, and should be allowed to assume the truth of many propositions. He held that the teacher must recognize that boys take unkindly to abstract reasoning. He criticised Oxford because, for the pass degree there, two books of Euclid must be memorized, even including the lettering of figures, no original exercises being required. In the discussion that followed, all favored the preliminary experimental course and some advocated a second experimental course to accompany Euclid. Hudson and Forsyth still believed in maintaining the Euclidean sequence of theorems. Minchin declared Euclid's order bad. S. P. Thompson and MacMahon favored the retention of Euclid. Miall did not see why we should have a recognized geometry any more than one arithmetic, or one trigonometry. Minchin, Magnus, Pressland, Workman, and Lamb declared themselves against Euclid as a text-book.

The immediate result of Perry's address of 1900, at Glasgow, was the appointment of two committees, one of the British Association and the other of the Mathematical Association. The former committee confined its work to the more general aspects of geometrical teaching. The latter, which was composed mainly of schoolmasters, formulated a set of detailed recommendations, which were published in the *Mathematical Gazette* of May, 1902. They include an experimental introductory course, requiring the use of instruments, practical measurement and numerical work.

[42] *Discussion on the Teaching of Mathematics*, edited by John Perry, 1901, p. 97.

In the formal study of geometry is recommended the retention of Euclid as a framework, the admission of hypothetical constructions, definitions not to be taught *en bloc,* the omission of incommensurables in the ordinary school course, and the use of algebra in the treatment of areas.

The Perry laboratory method has led to the preparation of some severely practical works, but as Lodge says, Perry "overemphasized fact divorced from principles." A middle ground has met with greater favor. The plans recommended by the Mathematical Association have been embodied very successfully by Godfrey and Siddons in a text-book entitled, *Elementary Geometry, Practical and Theoretical* (Cambridge University Press, 1904). The recommendations of the Mathematical Association have met with favor among teachers, and the general effect has been beneficial. A circular issued in 1908-1909, by the Board of Education, on *The Teaching of Geometry and Graphic Algebra,* showed the wide departure made since the beginning of the twentieth century. We quote two sentences: "Axioms and postulates should not be learnt or even mentioned." "It should be frankly recognized that unless the power of doing riders has been developed, the study of the subject is a failure."[43]

The greatest obstacle to reform in England has been the system of examinations. After thirty years of failures the Mathematical Association, at last, has been remarkably successful in persuading examining bodies to give up their insistence upon Euclid, and now Euclid's proofs and arrangement are no longer required by the universities. "Any proof of a proposition will be accepted which appears to the examiners to form a part of a logical order of treatment."

THE UNITED STATES OF AMERICA.

During the seventeenth century, arithmetic and geometry received some attention in the last year of the college course at Harvard College. In 1726 Alsted's *Geometry* is mentioned as a text-book studied by Harvard seniors, but as soon as geometry came to receive serious attention in American colleges, Euclid became the text used. The first mention of Euclid that we have seen, at Yale, is in 1733; at Harvard, in 1737. In the latter part of the eighteenth century, geometry was taught to lower classmen. According to a member of the Harvard class of 1798, "the sophomore year gave us Euclid to measure our strength." In 1801

[43] *Nature*, Vol. 80, 1909, May 27, p. 374.

Professor Webber said, "A tutor teaches in Harvard College Playfair's Elements of Geometry."

In 1813 the "Analytical Society" was formed at Cambridge in England, which aimed to encourage in Britain the vigorous study of French higher mathematics. The influence of this movement reached the United States. In about ten years American teachers began to adopt French texts. Collateral events at West Point had the same tendency. There elementary mathematics was taught from 1808 to 1810 by F. R. Hassler, who was a graduate of the University of Berne in Switzerland. In 1817 Crozet, of the Polytechnic School in Paris, introduced descriptive geometry into West Point.

In 1819, John Farrar, of Harvard, brought out a translation of Legendre's Geometry, which, with translations made by him of other French and Swiss texts on mathematics, were at once widely adopted in the leading American colleges. American teachers were willing to turn to the French, not only for works on the calculus and celestial mechanics, but also for books on elementary mathematics. So it came about that Euclid was replaced by Legendre. In 1828 Charles Davies, professor at West Point, brought out an edition of Brewster's translation of Legendre's Geometry. Davies did not enunciate propositions with reference to and by the aid of the particular diagram used for the demonstration, and to that extent returned to the method of Euclid. Davies' edition became widely popular under the name of "Davies-Legendre," and was much used in the United States as late as the '70's.

One of the earliest American geometries worthy of note, was that of Benjamin Peirce. The Harvard catalogue of 1838 announces that Freshman take Peirce's Geometry. Peirce favored the use of infinitesimals and also the use of the term direction, a concept probably first used in this country by a Harvard teacher named Hayward in his geometry of 1829. Peirce's text did not become widely popular, for, like his other elementary books, it was too condensed for immature students. In 1843 or 1844, Harvard first made geometry a requirement for admission to College.

In 1851, Professor Elias Loomis, of Yale, issued a geometry which was revised in 1871. Loomis came under French influences as a student in Paris. In the second edition of his text he says: "The present volume follows substantially the order of Blanchet's Legendre, while the form of the demonstration is

modeled after the more logical method of Euclid." It has been said of American writers, that while they have given up Euclid, they have modified Legendre's Geometry so as to make it resemble Euclid as much as possible. This applies to Loomis with greater force perhaps than to any other author.

In 1871 Professor Olney, of the University of Michigan, published a Geometry under two main heads:

I. *Special or Elementary Geometry,* comprising (1) Empirical Geometry, (2) Demonstrative Geometry, (3) Original Exercises in the Application of Algebra to Geometry, (4) Trigonometry.

II. *General Geometry* (Plane Loci).

Olney was a self-educated man. He was a great teacher and had original ideas about teaching. It is said that he was prevented by his publishers from departing very far from the traditional classification. His ideas were novel and forecasted in many ways the present tendencies in mathematical teaching. His geometry shows that he attempted to correlate the various mathematical topics and to introduce applications to everyday affairs. Olney's books were used quite extensively in the Middle West, but acquired no firm foothold in the East.

Just before the death of William Chauvenet, in 1870, appeared his *Geometry,* the only elementary book he wrote. Closely following French models, exhibiting a wonderful ease and grace of style, Chauvenet produced a remarkable book, which was used in many of the best schools. He included as a part of the work, an introduction to modern geometry. Perhaps no work on geometry ever published in the United States has been so highly respected as this.

In 1878 appeared the geometry of G. A. Wentworth, which is still in use. We omit all discussion of it, as also of later books which have been published in this country.

The researches on non-Euclidean Geometry, begun in the eighteenth century in Italy and Germany, and brought to fruition in the early part of the nineteenth century, did not produce appreciable effect upon the teaching of elementary geometry until the last quarter of the nineteenth century. It was in 1867 and 1868 that Baltzer, Battaglini, Grunert, and Hoüel brought Bolyai and Lobatchevsky to the attention of the mathematical public at large.

The new ideas have not affected the teaching of elementary geometry except in some of the definitions and postulates. They have assisted in the rejection of the definitions, "parallel lines are

lines everywhere equally distant," and "parallel lines are straight lines which have the same direction." They have shown the futility of "proving" the parallel-postulate and have led to the use of the word "axiom," not as a "self-evident truth," but as a synonym for "postulate."

In conclusion, we note that, with the beginning of the twentieth century, England began once more to influence the teaching of geometry in the United States, through the so-called "Perry movement," and that Germany, which at no time during the nineteenth century affected geometrical teaching in America, makes itself felt at the present time through the pupils of Klein and Hilbert and through the international movement towards reform in the teaching of mathematics, headed by Klein.

(*To be continued in May and June.*)

OUTPUT AND VALUE OF THE PRINCIPAL METALLIC PRODUCTS IN 1909.

The output and value of the chief metallic products from domestic sources in the United States for the calendar year 1909 are shown in the subjoined table. The value of the pig iron produced has not yet been computed.

METALS.	QUANTITY.	VALUE.
Pig Iron (long tons)	25,795,471	
Gold (fine ounces)	4,821,701	$99,873,400
Silver (fine ounces)	54,721,500	28,455,200
Copper (pounds)	1,092,951,624	142,083,711
Lead (short tons)	354,188	30,460,168
Zinc (short tons)	230,225	24,864,300
Quicksilver (flasks)	21,075	888,710
Tungsten (short tons)	1,619	614,370
Antimonial Lead (short tons)	8,889	847,731

QUICKSILVER IN 1910.

Preliminary figures collected from the individual producers show that the production of quicksilver in the United States in 1910 was 20,937 flasks of 75 pounds each, valued, at the New York average price for the year, $47.06, at $985,295. This production represents the output as given by the largest producers for the first eleven months of the year and the sum of estimates made by them for December, and also the final figures of the smaller producers.

A comparison of the statistics of production given above for 1910 with the final figures for 1909, 21,075 flasks, valued, at the average New York price for 1909 ($46.30), at $975,773, shows that although there was a decrease in production for 1910 of 138 flasks, there was an increase in value of the output of $9,522.

The production of the United States for 1911 will probably show a further decrease, owing to suspension of operations at several large mines.

THE PASSENGER PIGEON INVESTIGATION.[1]

By C. F. Hodge,
Clark University, Worcester, Mass.

At the meeting of this Union a year ago a plan was projected and has since been developed to secure adequate search of the American continent for this lost species. If any consider this a "fool's errand," I may add that at that time I put the question fairly to the Union as a body and to a number of you personally: "Do you think that scientifically adequate search has been made for *Ectopistes migratorius?*" Not a decisive affirmative answer was offered, and, among others, Dr. C. Hart Merriam replied distinctly that he did not.

The new plan called for discovery and confidential and exclusive notification of undisturbed nesting pairs or colonies. Just on this point the newspapers have not been as careful as they should in stating that the rewards were offered for "nests." In consequence, of course, nests, fortunately of mourning doves, began coming to me by express.

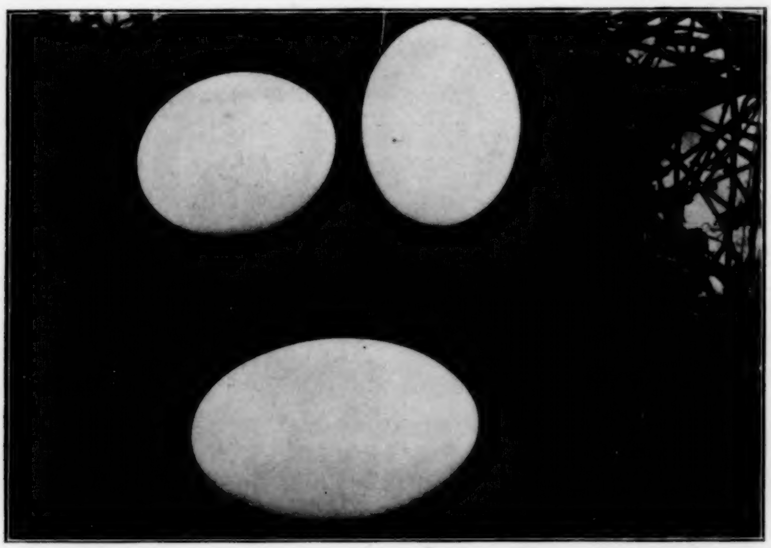
Since the investigation is undertaken solely with the purpose of discovering breeding pigeons *in time* to secure to them absolute protection, no such claim, however meritorious, could be treated as valid. The newspapers have contributed good service to the cause by disseminating notices, but, with three or four notable exceptions, the service might have been far greater, if their reporters had been clear headed enough to have grasped this essential purpose of the investigation. I must, however, bespeak added care in stating the one essential point that all rewards are offered for information of undisturbed *nesting pairs or colonies*—occupied nestings—of passenger pigeons. Not a reward can be paid for anything else.

It was, of course, expected a year ago that one season's search would settle the matter definitely one way or the other. However, negative evidence is proverbially inconclusive. No time limit has been set in any of our announcements so that, in order to keep faith with the public, we must let the awards stand for at least one more season. The investigation is to close definitely on October 1, 1911, and all offers of rewards not claimed by that date will be called off. With the general stirring up the matter has had this year, the observations of another season ought to settle the case for all time.

[1]Read at the meeting of the American Ornithologists' Union held in Washington, D. C., November 14-17, 1910.

In identifying the birds the size of the egg—and, almost certainly, the number of eggs or squabs in a nest—will prove one of the most important distinguishing features. The late Professor Whitman wrote, April 13, 1910:

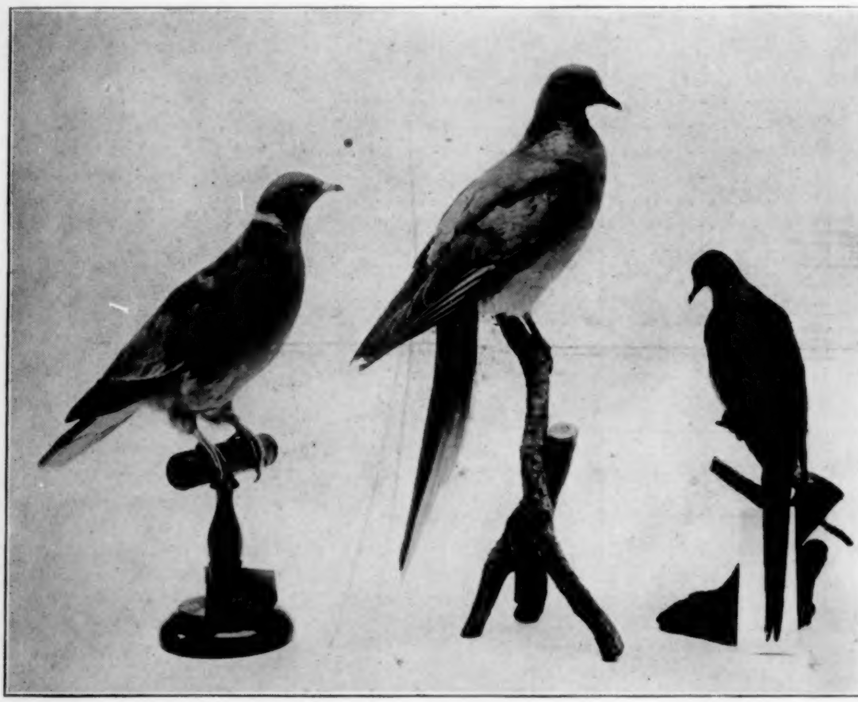
"One point you ought to emphasize in your circular, namely, The passenger pigeon NEVER lays TWO eggs. That point is absolutely certain. If you have a nest of two eggs, it is the nest of mourning doves. The egg hatches in 12½ days and the young flies from nest in 14 days. Thus one must not lose time in verification."



The two smaller of these eggs are those of the mourning dove and are of the exact size and shape. The largest is the passenger pigeon egg of exact size and shape. The nest is that of the mourning dove.

Again, in reply to my query as to how Audubon could have been mistaken he writes April 27, 1910: "As to *one* egg I am certain. The birds were bred in England, and they laid but one egg as did mine *invariably*. Audubon says they lay two, but he evidently thought all pigeons lay *two* and therefore that the passengers do the same. But it is now known that several American species lay but one egg: e. g., C. rufina, C. maculosa, C. fasciata (band-tail), etc. Some collectors may have occasionally found 1 to 3 eggs in a nest, but, if so, it must have been a case of more than one hen laying in same nest. If a hen had no mate, she would steal the opportunity, as I saw one of mine do."

All the nests I was asked to verify last summer had two eggs and were nests of mourning doves. It will be only under exceptional conditions that I shall incur expense or loss of time investigating nests with two eggs or squabs in them this season. The pigeon's egg is, roughly, 1½ inches long, that of the dove is about one inch.



BAND-TAILED PIGEON	PASSENGER PIGEON	MOURNING DOVE
Columba fasciata	*Ectopistes migratorius*	*Zenaidura macroura*
Stocky form, square tail, west of Rocky Mountains	Length 10 inches, back blue-gray, breast ruddy	Length 12 inches, distinguished from pigeon by black spot on side of neck

(Photographed by courtesy of American Museum of Natural History.)

The Band-Tailed Pigeon in the West and the Mourning Dove in the East cause practically all the confusion, and unnecessary correspondence.

Although we have no tangible evidence to show, the testimony for the season is greater in amount than for any one year since 1900. After sorting out the season's correspondence I have as possibly true for pigeons seen during 1909 and 1910 reports as follows: From Ontario, 10; from Pennsylvania, 9; from Massachusetts, 8; from New York, 4; from Michigan, 3; from Iowa, 2; and from Illinois, Nebraska, Wisconsin, Minnesota, Connecticut, New Hampshire, and Manitoba, one each.

As a last hope it was thought that these reports might be made to support one another by indicating, when plotted on a map, a consistent course of one or more flocks of pigeons migrating over the continent. So few of these reports, however, give definite data as to number in flock or direction of flight that even this slight hope finds little or no support. The absence of any definite word from reputable naturalists or ornithologists, American or Canadian, is another extremely discouraging and ominous feature of this season's investigation. Thus what on the surface appear to be the most encouraging reports, in which flocks of hundreds or even thousands of the pigeons have been seen, forfeit all credence on account of their isolation. If, as one report states, the pigeons were nesting in "almost countless numbers"—somewhere in northern United States or Canada (I do not wish to know where until informant is able to report occupied nestings during the present season), it seems utterly incredible that, in flying to or from their roost, either in feeding or in migrations, they should not have been seen by many others. Another discouraging feature of this year's work is the total absence of reports from the Southern States. If flocks of pigeons are in existence, they ought to be most easily and frequently observed in their southern winter range. Except for unlikely reports from the mountains of western Texas, the Southland has been one vast silence the entire season.

It now looks as if the worst fears of American naturalists were about to be confirmed and that we are "in at the death" of the finest race of pigeons the world has produced.

In the campaign of this season it is proposed to appeal especially to the students and school children, college and university, high school and public school, of this country and Canada, making it thus distinctively educational. Then the awakening to the problem, the bird study necessary to enable one to identify pigeons, if discovered, the wholesome field work will each be worth vastly more to the country at large than the mere cost of this investigation, even if no pigeons are discovered. A number of other valuable species demand united and effective protection, if they are to be saved from a like fate.

It is to be hoped that renewed effort for another year may result in what may be generally considered an adequate search of the continent for *Ectopistes migratorius.*

List of Rewards with Conditions Governing Them.

One Thousand Dollars ($1,000) Reward.

For first information, exclusive and confidential, of the location of a nesting pair or colony of passenger pigeons, anywhere in North America; when properly confirmed and if found by confirming party with parent birds and eggs or young undisturbed:

Colonel Anthony R. Kuser will pay a reward of $300.
John E. Thayer will pay a reward of..............$700.

For first nesting discovered thereafter in the following states will be paid by:

John Burroughs, New York	$100
A. B. F. Kinney, Massachusetts	100
Anonymous, Massachusetts, for 2d find	100
Allan B. Miller, for 1st nesting found in Worcester Co., Mass.	20
Edward Avis, Connecticut	100
Harry S. Hathaway, Rhode Island	100
Worthington Society, New Jersey	100
John Dryden Kuser, for 2d nesting found in New Jersey	10
Henry W. Shoemaker, Penna. $200 (adds $25, if nest is protected)	225
W. B. Mershon, Michigan	100
R. W. Mathews, Minnesota	100
Ruthven Deane, Illinois	50
John E. Thayer, Me., N. H., Vt., Ont., Wis., $100 each	500
John Lewis Childs, for first three nestings not entitled to any of the above rewards, $200 each	600

The purpose of these offers is to secure an intelligent search of the American continent for breeding pigeons in the hope that, if found, the species may be saved from extermination.

All above rewards are offered solely and only for information of location of undisturbed nestings. We do not desire possession of any birds, alive or dead, but are working solely to save the free, wild pigeon.

To insure intelligence and good faith informants of nestings are advised to enclose or agree to forfeit at least $5.00 in case they have failed to identify the birds correctly. This is only fair, since the amount may cover but a small part of the costs occasioned by a false report. The money will be immediately returned, if the birds are found to be passenger pigeons (*Ectopistes migratorius*). In the case of nesting pigeons, there can be no excuse for sending in false reports. Disregard all nests on the ground. The wild pigeon always nests in trees, generally ten feet or more from the ground.

Priority of claim will be decided by time of receipt at post or telegraph office. Rewards will be equally divided, if two or more letters or messages bear record of same date and hour. All nestings within one mile of one another will be counted as one colony.

Please report all pigeons seen, giving exactly date, hour, number in flock, direction of flight. Unless absolutely certain that you know the Band-tailed, Viosca, and Red-billed pigeons, do not report that you have seen the passenger pigeon in the Rocky Mountains or Pacific Coast region, from British Columbia to Mexico.

As soon as a pigeon nesting is surely identified write to C. F. Hodge, Clark University, Worcester, Mass. (enclose postage for reply), who will arrange for confirming party and for payment of the reward. All rewards not claimed by October 31, 1911, will be withdrawn.

For descriptive leaflet with colored pictures of the pigeons and mourning dove, enclose six cents, stamps, to Chas. K. Reed, Worcester, Mass.

Although standing practically as above during the entire season of 1910, no rewards were successfully claimed. This makes the outlook very dark for saving the finest race of pigeons the world has ever evolved. Still reports of pigeons observed in the Eastern and Middle States during 1910 afford some encouragement that nestings may be found in 1911. The work is to be pushed in the schools the coming season, thus making the effort educational, and it is hoped that some teacher or pupil will find the nestings and secure the rewards.

If pigeons are found anywhere, it is thought that feeding them liberally with buckwheat or other grains may induce them to nest in the neighborhood. Salt is also a great attraction.

Theodore Roosevelt has expressed his desire to join the confirming party, if certain that a nesting has been discovered; and it is proposed to organize about such a find the Passenger Pigeon Restoration Club of America to protect the birds effectively over the entire continent.

WELL PROTECTION TO PREVENT TYPHOID FEVER.

The menace of typhoid fever in country districts—a menace arising from polluted drinking water—is thoroughly considered in a report by Myron L. Fuller of the United States Geological Survey in which the various sources of pollution are indicated and suggestions are given for means of protection.

Typhoid fever rates are usually greater in the country than in cities, despite the prevailing belief that farms, isolated as they are from areas of congested population, are ideally situated for obtaining pure and wholesome water.

Failure to protect adequately the wells in farming districts is given as the most common reason for their pollution, and ignorance of the manner in which ground water circulates is the cause of the faulty protection. Chemical analysis is not rated high by Mr. Fuller as a means of detecting polluted water, for he asserts that a careful common sense inspection of the district is usually much more to the point.

Sources of pollution in the vicinity of a well or spring should be noted wherever possible, and drinking water should not be drawn except at a safe distance from them. The distance required for absolute safety varies greatly with the character of the rock. For wells sunk in sandstone, slate, and shale, 100 feet may be sufficient; where the surface stratum is composed of fine sand 200 feet should be allowed; and where it is limestone much greater distances will be necessary. Water may run polluted in limestone for miles, so that wells in regions where limestone makes up the greater part of the surface rock should be carefully examined after rains for mud and floating matter, for these are pretty sure indications of pollution.

For protecting wells, springs, and cisterns Mr. Fuller advocates, first of all, a water-tight lining to keep out surface water. Wells and springs should always be covered and protected from animals, dust, and falling leaves. Watering troughs should always be located a safe distance away, though the custom prevails in country districts of having well and trough side by side.

Mr. Fuller's report is printed as Water Supply Paper 255, which may be obtained free by applying to the Director U. S. Geological Survey, Washington, D. C.

PROBLEM DEPARTMENT.

By E. L. Brown,

Principal North Side High School, Denver, Colo.

Readers of this magazine are invited to send solutions of the problems in which they are interested. Problems and solutions will be duly credited to their authors. Address all communications to E. L. Brown, 3435 Alcott St., Denver, Colo.

Algebra.

234. *Proposed by E. B. Escott, Ann Arbor, Mich.*

Solve $xy + zu = 17$ (1)

$yz + ux = 11$ (2)

$xz + yu = 13$ (3)

$x + y + z + u = 11$ (4)

(Rietz and Crathorne's College Algebra, p. 79, ex. 31.)

Solution by J. A. Whitted, Abingdon, Ill., and C. F. Northrup, Waterbury, Conn.

From (1) and (2) by adding and factoring,

$$(x + z)(y + u) = 28 \qquad (5)$$

Squaring (4) and subtracting 4 times (5),

$$(x + z)^2 - 2(x + z)(y + u) + (y + u)^2 = 9$$

$$\therefore x + z - y - u = \pm 3 \qquad (6)$$

Similarly,

$$x + u - y + z = \pm 1 \qquad (7)$$

$$\text{and } x + y - z - u = \pm 5 \qquad (8)$$

From (4), (6), (7), and (8) we get 8 sets of equations linear in x, y, z, and u. Solving these we get the following values:

$x=5, \frac{9}{2}, \frac{7}{2}, 3, \frac{5}{2}, 2, 1, \frac{1}{2}$

$y=3, \frac{7}{2}, \frac{9}{2}, 5, \frac{1}{2}, 1, 2, \frac{5}{2}$

$z=2, \frac{5}{2}, \frac{1}{2}, 1, \frac{9}{2}, 5, 3, \frac{7}{2}$

$u=1, \frac{1}{2}, \frac{5}{2}, 2, \frac{7}{2}, 3, 5, \frac{9}{2}$.

235. *Proposed by H. C. Fetsch, Cincinnati, O.*

Sum to n terms the series,

1, 1, 2, 3, 5, 8, 13, 21, 34, etc.

I. Solution by Richard Morris, New Brunswick, N. J.

In the series $1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 + 34 +$, etc., in which any term equals the sum of the two preceding terms, the sum of n terms is equal to one less than the $(n + 2)$th term.

Assume $(n + 2)$th term $- 1 =$ sum of n terms.

Add the $(n + 1)$th term to both members.

The left member becomes $(n + 2)$th term $+ (n + 1)$th term $- 1$.

But by the law of formation of the series, the sum of the $(n + 2)$th and $(n + 1)$th terms equals the $(n + 3)$th term. The right member becomes the sum of $(n + 1)$ terms. Thus, $(n + 3)$th term $- 1 =$ sum of $(n + 1)$ terms.

Hence, if the theorem holds true for a value n, it holds true for the next greater value of n. But it does hold true for small values of n, as can easily be verified.

It is true when $n = 7$, $\therefore$ it is true when $n = 8$.

Similarly when $n = 8$, and when $n = 9$, etc.

Therefore it is true for all values of n.

II. Solution by E. G. Berger, Hutchinson, Minn.

This is a series in which every term is the sum of the two preceding

terms. Let k represent the $(n-2)$th term, l the $(n-1)$th term; the nth term is then $k+l$. The series is then, to n terms:

$$1, 1, 2, 3, 5, 8, 13, 21 \ldots\ldots k, l, k+l.$$

Since every term is the sum of the two preceding terms, every term is also the difference of the two following, and the series may be written as a series of differences as follows:

$$2-1,\ 3-2,\ 5-3,\ 8-5,\ 13-8,\ 21-13 \ \ldots\ldots l-k,\ (k+l)-l,\ k+l.$$

The nth and the $(n-1)$th term having no succeeding terms cannot be expressed as such differences.)

Taking the sum of the series in this form the first number in every term is seen to cancel the second number in the next following term, leaving uncancelled (or, in other words, as the sum of the series) the second number of the first term, the first number of the $(n-2)$th term, the $(n-1)$th term and the nth term; i. e.

$$-1+(k+l)+l+(k+l) \text{ or } 2(k+l)+l-1$$

Therefore the sum, to n terms, of a series in which each term is the sum of the two preceding terms is equal to twice the last, or nth term, plus the $(n-1)$th term minus the second term.

III. Solution by I. L. Winckler, Cleveland, O.

The series $1, x, 2x^2, 3x^3, 5x^4, 8x^5, \ldots\ldots$ is a recurring series, the scale of relation being $1-x-x^2$.

The sum of this series to infinity is $\dfrac{1}{1-x-x^2}$

$$=\frac{1}{\sqrt{5}}\left\{\frac{1}{\frac{1}{2}(\sqrt{5}+1)+x}+\frac{1}{\frac{1}{2}(\sqrt{5}-1)-x}\right\}$$

$$=\frac{1}{\sqrt{5}}\left\{[\tfrac{1}{2}(\sqrt{5}+1)+x]^{-1}+[\tfrac{1}{2}(\sqrt{5}-1)-x]^{-1}\right\}$$

Expanding each term in the bracket by the Binomial Theorem, we find for the nth term, $\dfrac{1}{2^n\sqrt{5}}\left\{(-1)^{n-1}(\sqrt{5}-1)^n+(\sqrt{5}+1)^n\right\}x^{n-1}$

Now $S=2u_n+u_{n-1}-1$ from the usual formula for the sum of a recurring series where $S=$ sum of n terms, and $u_1, u_2, \ldots\ldots u_n$ the series, and $x=1$.

$$\therefore S=\frac{1}{2^{n-1}\sqrt{5}}\left\{(-1)^{n-1}(\sqrt{5}-1)^n+(\sqrt{5}+1)^n+(-1)^{n-2}(\sqrt{5}-1)^{n-1}\right.$$
$$\left.+(\sqrt{5}+1)^{n-1}\right\}-1$$

Geometry.

236. *Proposed by A. Heinz, Savannah, Mo.*

Prove that the bisectors of the vertex angles of all triangles on the same side of a common base meet in a point when, *and only when*, these vertex angles are equal.

Solution by H. E. Trefethen, Kent's Hill, Me.

Draw an indefinite line AA′ perpendicular to the given base BB at its middle point K, A′ being on the same side of BB as the required triangles.

(1) Let P be any point in AK. Describe the circle BPB and from P draw chords meeting the opposite arc in points which are vertices of triangles all inscribed in the same circle and having equal vertex angles. The chords through P are their bisectors.

(2) Let P be any point on either side of BB. If P is on the same side of BB as A′ and between lines through B and B parallel to AA′, through points anywhere in AK as Q, R, S, etc., describe the circles BQB, BRB, etc. Through PQ, PR, etc., draw chords (or secants) cutting the oppo-

site arcs respectively in D, E, F, etc. The triangles BDB, BEB, BFB, etc., all have different circumcircles and hence unequal vertex angles whose bisectors all pass through P.

If P is not between the parallels, produce PB to meet AK at X. The points Q, R, S, etc., must all lie between K and X in order to give a solution.

If P is on the other side of BB and between the parallels, draw PB cutting AK in Y and produce BP meeting AK at Y′. Points Q, R, S may now lie anywhere in AK except between YY.′ But if P is not between the parallels, Q, R, S must be between Y and K.

237. *Proposed by R. F. Holden, Kalamazoo, Mich.*

Find the locus of all points from which two unequal circles subtend equal angles.

I. Solution by I. L. Winckler, Cleveland, O.

Let O and C be the centers, P any point in the required locus.

Draw PA and PB tangents to circle with center C and PE and PD tangents to circle with center O.

Draw radii OE and BC.

△ POE is similar to △ PBC.

∴ OP : CP = OE : BC.

∴ The locus of P is the locus of a point whose distances from O and C are in the constant ratio OE : BC.

To find the locus divide OC internally at F, and externally at G in the ratio OE : BC, and on FG as a diameter draw a circumference. This circumference is the required locus.

II. Solution by H. E. Trefethen, Kent's Hill, Me.

Let the centers be, O of the larger circle, O′ of the smaller, and P any point of the required locus. Draw the exterior tangents meeting at C, the interior meeting at C′. P, C and C′ are points from which the circles are seen under the same angle. Hence by similar triangles R′/R = O′P/OP = C′O′/C′O = CO′/CO. Hence PC′ bisects the angle OPO′ and PC bisects the exterior adjacent angle. Therefore the angle CPC′, formed by the bisectors of two pairs of vertical angles, is a right angle and its vertex P is on the circumference of a circle whose diameter is CC′.

238. *Proposed by Mary S. Sabin, Denver, Colo.*

An approximate construction of a regular polygon of n sides is as follows: Divide the diameter AB into n equal parts, $AA_1 = A_1A_2 = A_2A_3 \ldots\ldots = 2R \div n$. With A and B as centers and AB as a radius describe arcs intersecting in C. Produce the line segment CA_2 to intersect the circle in D. Then will AD be approximately a side of the regular polygon. Find the actual value of AD expressed by R and n, and show in which polygon or polygons the above construction is absolutely correct.

I. Solution by H. E. Trefethen, Kent's Hill, Me.

Let O be the center of the circle and CD cut AO in A_2. Draw DF perpendicular to AO. Put $AA_2 = 4R/n$, $OA_2 = R(n-4)/n$, $OC = R\sqrt{3}$, $FB = y$, $AF = 2R - y$, $FA_2 = y - 2R + 4R/n$, $DF = z$. Then

$DF/FA_2 = OC/OA_2$. Hence $z = \sqrt{3}\,(ny - 2nR + 4R)/(n-4) \ldots.$ (1)

Also $z^2 = y(2R - y) \ldots.(2)$ and $AD^2 = 2R(2R - y) \ldots.$ (3)

From (1) and (2) $z^2 = 3(ny - 2nR + 4R)^2/(n-4)^2$. Whence

$y = R\left[7n^2 - 20n + 16 + (n-4)\sqrt{n^2 + 16n - 32}\right]/(4n^2 - 8n + 16)$. Hence by (3)

$AD^2 = R^2\left[n^2 + 4n + 16 - (n-4)\sqrt{n^2 + 16n - 32}\right]/(2n^2 - 4n + 8)$.

When $n=3$, $AD = R\sqrt{3}$; when $n=4$, $AD = R\sqrt{2}$; when $n=6$, $AD = R$.

Thus in these three cases the results are absolutely correct.

When

$n=5$, $AD=R\sqrt{(61-\sqrt{73})/38}=1.174913R$; $a=R\sqrt{(5-\sqrt{5})/2}=1.175570R$.

$n=8$, $AD=R\sqrt{(14-\sqrt{40})/13}=0.768387R$; $a=R\sqrt{2-\sqrt{2}}=0.765367R$.

$n=10$, $AD=R\sqrt{(13-\sqrt{57})/14}=0.623937R$; $a=R(\sqrt{5}-1)/2=0.618034R$.

$n=12$, $AD=R\sqrt{(26-\sqrt{304})/31}=0.525615R$; $a=R\sqrt{2-\sqrt{3}}=0.517638R$.

$n=7$, $AD=0.869177R$; $a=0.867767R$.

II. Solution by Editor.

$$AA_2=\frac{4R}{n},\ A_2O=\frac{(n-4)R}{n},\ CO=R\sqrt{3},\ CA_2=\frac{2R}{n}\sqrt{n^2-2n-4}.$$

$$\text{Sin }\phi=\frac{CO}{CA_2}=\frac{n\sqrt{3}}{2\sqrt{n^2-2n+4}}.\quad \text{Let } AD=x_1 \ \cos\alpha=\frac{x_1}{2R}.$$

$$\text{Sin }\beta=\sin 2\alpha=2\sin\alpha\cos\alpha=\frac{x_1}{2R^2}\sqrt{4R^2-x_1^2}.$$

$$A_2D^2=AA_2^2+AD^2-2AA_2\cdot AD\cos\alpha=\frac{(n-4)x_1^2}{n}+\frac{16R^2}{n^2} \tag{1}$$

$$\text{Also }\frac{A_2D^2}{OD^2}=\frac{\sin^2\beta}{\sin^2\phi}\quad \therefore A_2D^2=\frac{(n^2-2n+4)(4R^2x_1^2-x_1^4)}{3n^2R^2} \tag{2}$$

From (1) and (2) we easily find

$$(n^2-2n+4)x_1^4-(n^2+4n+16)R^2x_1^2+48R^2=0$$

$$\therefore x_1=R\left[\frac{n^2+4n+16\pm(n-4)\sqrt{n^2+16n-32}}{2(n^2-2n+4)}\right]^{\frac{1}{2}} \tag{3}$$

Another Method.

Prolong OB to E and OL to M so that $BE=LM=AA_1$.

Let $BA_{n-3}=3AA_1$. Join H and A_{n-3}. Then will HA_{n-3} be approximately a side of polygon of n sides.

Let $\angle HOB=\theta$ and $HA_{n-3}=x_2$.

In $\triangle HOE$, $OH/OE=\sin 45^\circ/\sin(45^\circ+\theta)$. But $OH=R$, and $OE=R+2R/n$.

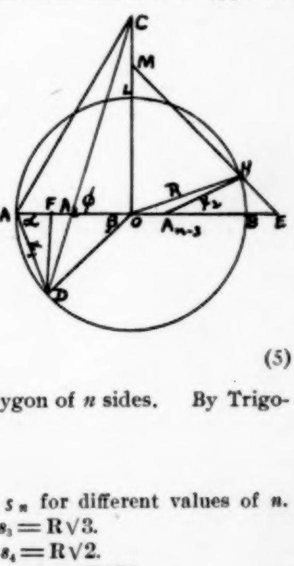
$\therefore R/(R+2R/n)=\sin 45^\circ/\sin(45^\circ+\theta)$, whence $\sin\theta+\cos\theta=1+2/n$.

$\therefore \cos\theta=\frac{1}{2}\left[1+2/n+\sqrt{1-4/n-4/n^2}\right]$.

In $\triangle$ O H A_{n-3}, $X_2^2=R^2+OA^2_{n-3}-2R\cdot OA_{n-3}\cos\theta$. (4)

$OA_{n-3}=R-6R/n$.

Substituting for $\cos\theta$ and OA_{n-3} in (4) and solving for x_2, we have



$$x_2=\frac{R}{n}\left[n^2-8n+48-(n-6)\sqrt{n^2-4n-4}\right]^{\frac{1}{2}} \tag{5}$$

Let s_n be one side of a regular inscribed polygon of n sides. By Trigonometry we have

$$s_n=2R\sin\frac{\pi}{n}.$$

Following are corresponding values of x_1, x_2, s_n for different values of n.

$n=3$	$x_1=R\sqrt{3}$.	x_2 is imaginary.	$s_3=R\sqrt{3}$.
$n=4$	$x_1=R\sqrt{2}$.	x_2 is imaginary.	$s_4=R\sqrt{2}$.
$n=5$	$x_1=1.1749R$.	$x_2=1.16619R$.	$s_5=1.17558R$.
$n=6$	$x_1=R$.	$x_2=R$.	$s_6=R$.
$n=7$	$x_1=0.86917R$.	$x_2=0.86752R$.	$s_7=0.86776R$.
$n=8$	$x_1=0.76838R$.	$x_2=0.76561R$.	$s_8=0.76536R$.

$n=10$ $x_1=0.62393R.$ $x_2=0.61698R.$ $s_{10}=0.61804R.$
$n=12$ $x_1=0.52561R.$ $x_2=0.51674R.$ $s_{12}=0.51764R.$
$n=16$ $x_1=0.40036R.$ $x_2=0.38974R.$ $s_{16}=0.39018R.$
$n=20$ $x_1=0.32378R.$ $x_2=0.31277R.$ $s_{20}=0.31286R.$

The first method is correct for $n \doteq 3, 4, 6$, and the second for $n=6$. Comparing the values of x_1 and x_2 with those of s_n which are correct to five decimal places, we note that x_1 becomes more and more incorrect as n increases, while x_2 becomes more nearly correct.

For the triangle and the square the expression for the second method assumes the imaginary form, shown graphically by the line EF not meeting the circle.

A *general* solution of this problem within the limits of Euclidean geometry is just as impossible as that of equations of higher degrees. Trigonometry furnishes the solution of the general to any degree of precision.

Credits for Solutions Received.

Algebra 221—Artemus Martin. (1)
Algebra 229—O. K. Lie (2 solutions). (2)
Geometry 232—G. I. Hopkins, O. K. Lie (2 solutions). (3)
Algebra 234—E. G. Berger, O. L. Brodrick, A. L. McCarty, C. F. Northrup, C. A. Perrigo, H. A. Porter, Kate Schweibold, J. M. Townsend, H. E. Trefethen, J. A. Whitted, H. C. Willett, I. L. Winckler. (12)
Algebra 235—E. G. Berger, R. Morris, H. E. Trefethen, I. L. Winckler. (4)
Geometry 236—O. L. Brodrick, C. A. Perrigo, H. E. Trefethen, I. L. Winckler. (4)
Geometry 237—H. E. Trefethen, I. L. Winckler. (2)
Geometry 238—H. E. Trefethen, I. L. Winckler, The Editor. (3)
Total number of solutions, 31.

PROBLEMS FOR SOLUTION.

Algebra.

244. *Proposed by A. B. Carlton, Walla Walla, Wash.*

Find the value to n terms of the continued fraction

$$\cfrac{x}{1+\cfrac{x}{1+x \; \cdots}}$$

245. *Proposed by Norman Anning, Chilliwack,* B. C.

Find the general term of

0, 0, 1, 1, 2, 4, 5, 7, 10, 12, 15, 19, ...

Geometry.

246. *Proposed by Orville Price, Denver, Colo.*

Show that any two perpendicular lines terminated by the opposite sides of a square are equal to one another, and by this property show how to escribe a square to a given quadrilateral.

247. *Proposed by H. E. Trefethen, Kent's Hill, Me.*

If the incircle passes through the centroid of the triangle, find the relation between the sides a, b, c.

248. *Selected.*

If through a point O within a triangle ABC parallels EF, GH, IK to the sides be drawn, the sum of the rectangles of their segments is equal to the rectangle contained by the segments of any chord of the circumscribing circle passing through O.

REPORT OF THE MINNEAPOLIS MEETING OF THE AMERICAN FEDERATION OF TEACHERS OF THE MATHEMATICAL AND THE NATURAL SCIENCES.

The annual meeting of the Council of the American Federation was held at the University of Minnesota, Minneapolis, Minn., on Wednesday morning, December 28, at 10 A. M.

The roll call of the meeting showed seven delegates present, six from the Central Association of Science and Mathematics Teachers and one from the Association of Mathematical Teachers in New England. Since there was not a quorum present, it was announced that all business transacted would have to be ratified by written vote of the Council.

In the absence of the secretary, Professor F. L. Charles of the University of Illinois was elected secretary pro tem.

The minutes of the last meeting (Boston, 1909) having been printed, it was voted that their reading be dispensed with and that they be approved as printed in SCHOOL SCIENCE AND MATHEMATICS for April, 1910 (Vol. X, p. 343).

The report of the secretary-treasurer was presented in print in Bulletin III, and it voted that the report be approved as printed.

Reports were presented from the Executive Committee, the Committee on College Entrance Requirements in Mathematics and Science, the Committee on Logarithmic Tables, the Committee on Bibliography, the Committee on a Syllabus in Geometry, the Committee on a Mathematical Journal, and the Committee on the Unit in Chemistry. It was voted that these reports be received, printed, and distributed to all members of the federated associations, in order that the Council and the associations might have opportunity to consider them before taking action on them. These reports are appended.

Reports were presented from six of the local associations. It was voted that these reports be printed and distributed. Reports are appended.

The nominating committee reported nominations for the officers of the Federation for the coming year as follows: President, C. R. Mann, the University of Chicago; secretary-treasurer, E. R. Smith, Polytechnic Preparatory School, Brooklyn, N. Y.; members of the Executive Committee—Ira M. DeLong, University of Colorado; I. N. Mitchell, State Normal School, Milwaukee; Wilhelm Segerblom, Phillips Exeter Academy, Exeter.

It was unanimously voted that the secretary cast the ballot for the nominees. The secretary cast the ballot, and those nominated were declared elected officers for the coming year.

On Thursday morning, December 29, the Federation held a joint session with Section L, A. A. A. S. This meeting was well attended. The topic, "Testing Results of Teaching Science," was presented by Professor E. L. Thorndike; and Professor O. W. Caldwell described some experiments in testing that were in progress at the University High School in Chicago. Professor Thorndike's paper is printed in full on page 315 of this magazine, and Professor Caldwell's will be printed as soon as the experiment is completed and the results ready for publication.

F. L. CHARLES, *Secretary pro tem.*

Since the meeting at Minneapolis, the actions there taken have been submitted to the Council by letter and ratified by written vote of the majority of the members. E. R. SMITH, *Secretary.*

Report of the Executive Committee.

In presenting its report this year, your executive committee wishes first to review briefly the activities of the Federation. The first preliminary meeting of delegates for the purpose of organizing this body was held in December, 1906. No organization was completed that year, but a committee was appointed to frame articles of federation and report in 1907. The articles of federation were presented to the delegates from the local associations at a meeting in Chicago in January, 1908. Before the organization could be completed, however, these articles had to be approved by each of the local associations. The year 1908 was spent in making the purpose of the Federation clear to the local associations and in securing the approval of seven of them to the articles of federation. These seven associations paid dues that year to the amount of about $70.00, of which $50.00 was spent in paying for the printing and postage used in the work of organization up to November, 1908.

In November, 1908, the executive committee issued an eight-page bulletin, which was sent not only to all the members of all the associations that had joined the Federation, but also to the members of those associations that were considering joining. This bulletin contained a brief statement of the proposed purpose and policy of the Federation, a brief history of its organization, the announcement of the appointment of a committee to prepare a bibliography of science teaching, and the call for the annual meeting in Baltimore in December, 1908. The cost of printing and distributing this bulletin was about $65.00.

Thus the work of making known what the Federation was for, and of interesting associations in it, occupied two full years, and cost $115.00. The work of advancing science teaching, for which the Federation was formed, could not begin until this preliminary work was completed.

During the year 1909 six more associations joined the Federation. A second bulletin, of twenty-four pages, containing the report of the meeting in Baltimore, the papers read there, together with the announcement of the appointment of committees on publication, on a syllabus in geometry, and on the relations between the high school and the college was issued that year. In the appointment of these committees the real work of the Federation began.

This second bulletin was again distributed to all members of all associations which had joined the organization, and was given wide publicity in the educational press. The total cost of printing and postage was about $75.00. Your committee considered it necessary to print and distribute these bulletins in this way, because there is as yet no established means of publication which reaches all the members of all the associations. The establishment of such a means of communication is still one of the problems before the Federation. Two committees are at work on it—the one on publications and the one on a mathematics journal.

A third bulletin of four pages has just been issued, containing the call to the meeting at Minneapolis, the announcement of the appointment of the committees authorized at the Boston meeting, the financial statement, and the list of associations which are now members of the Federation.

During the first two years, 1908-09, dues were paid by the associations at the rate of five cents per member. In return for this each member received Bulletins 1 and 2, beside assisting in paying the expenses necessary to the organization of so cosmopolitan a body. Last year the associations voted to raise the dues to ten cents per member, as it was clear that the work of the committees that had been appointed could not be supported and their reports printed without the additional income which resulted

from the increase in dues. Some of this money has been used in paying for printing and postage for the committees, but the greater part has been reserved for use in securing copies of the report of the committee on bibliography. This report has been completed and accepted by the U. S. Bureau of Education for issue as one of its special bulletins on education. Arrangements have been made to secure enough reprints of this report to send a copy to every member of each of the local associations in the Federation. Requests for copies of this report have been received from England, Germany, India, the Philippines, and Honolulu.

The committee on the relations between colleges and high schools is presenting an important report at this meeting. This report is the result of eighteen months of hard work on the part of this committee. It contains valuable suggestions as to the reorganization of those relations so as to make the conditions for science work in the schools more advantageous, and calls for a discussion of these suggestions by the local associations. This report should be printed and distributed as soon and as widely as possible, and the committee should be continued in order to follow up their suggestions and try to get them put into operation in as far as they are approved. As a result of his activities as chairman of this committee, Mr. Butler has been made a member of a committee of the Department of Secondary Education of the National Education Association, which has just been appointed to study this same problem. In this way the members of the Federation secure direct representation in the discussions of the N. E. A. on this matter.

The Committee on Geometry Syllabus has made great progress, and its report promises to be the best piece of work that has yet been done on this subject. The committee expects to finish its work during the coming year, and, when done, the report should be printed and distributed. Another committee is working on the Chemistry Unit, but this committee is not yet ready to report.

As a new topic worthy of consideration by the Federation, the executive committee this year proposes the problem of methods of testing the results of teaching. In order to introduce this topic, Professor Judd was invited to give us some suggestions last year, at the Boston meeting, and Professors Thorndike and Caldwell will give us further ideas on this matter at this meeting. The executive committee believes that no greater benefit can be conferred on a teacher than to interest him in his teaching problems and to encourage him in the scientific study of them. With this idea in mind the Bibliography of Science Teaching was the first piece of work undertaken by the Federation; and the papers presented at the meetings in Baltimore and Boston were planned to point in the same direction. Your committee now plans to follow with suggestions as to scientific methods of testing results, since on such methods depends the teacher's ability to find out definitely where improvement is possible.

Another line of investigation which the executive committee wishes to suggest to the Federation is that of studying the relations of each particular science to the field of science teaching as a whole, including the problem of the general science course and that of four year science courses in high schools. It is evident that progress in the solution of these problems must precede progress in the solution of the problems of each particular science, and that the Federation is in a far better position to carry on such study than is any one of the local associations by itself. We recommend this problem to the incoming executive committee as one to which the energies of the Federation might be directed to advantage.

In a nut shell: The Federation has been in existence for four years. It has cost the local associations not quite $400.00. It has issued three bulletins, brought about better acquaintance and understanding among science and mathematics teachers the country over, been a means of producing several valuable reports, and paved the way for securing for the secondary school teachers a voice in college entrance questions and an interest in the scientific study of their teaching problems.

Yet, notwithstanding the value of the work thus far accomplished, your executive committee believes that the work of the Federation is not yet fully understood by all the local associations. Experience has shown us that work of this kind must be done slowly, and the local associations are in many cases looking for quicker returns than is possible when so many interests are involved and everything has to be settled by correspondence. For example, nine months of constant correspondence among the members of this committee and the officers of local associations was required before this committee was able to settle the membership of the Committees on Relations between Colleges and High Schools and on a Syllabus in Geometry with the assurance that all the interests represented in the Federation were fairly well cared for. In like manner, five months of correspondence was required to select the members of the new Committee on Logarithmic Tables.

For the same reasons, the work of the committees appointed has been slow and laborious. That results have been attained at all is due to the persistent and extensive work done by the chairmen and the members of these committees. Because of this work, however, science teachers are beginning to be better acquainted with one another, and the difficulty of the work will surely diminish as time goes on. The hardest part of the work has now been done, and the Federation will be able with increasing efficiency to serve the cause of securing better conditions for teaching science, provided the local associations are ready to continue their cordial support of the organization. If such support fails, the work must grow more difficult and less effective.

During its term of service your committee has had very forcibly impressed upon it the fact that there are still many conflicting interests among the science teachers, and that work that must be carried on largely by correspondence must progress slowly. We are very sure that no other organization has yet produced reports more cosmopolitan and more suited to the needs of the times than those drawn up by the committees of the Federation. The value of these reports cannot be appreciated fully until they have been at work for some time. Unless the local associations are still firm in their belief that the Federation is being and will continue to be of great service to science teachers in enlarging and developing their common interests, we do not think that it is right to attempt to continue the organization and ask any five men to shoulder the work involved in membership in the executive committee. If the associations understand the work and encourage it, the work is well worth while.

Your executive committee therefore makes the following recommendations:

1. That this report and such other reports of committees as may be presented at this meeting be printed and distributed to all members of all associations as soon as possible.

2. That each association be requested to consider these reports very carefully, to consider the fact that the organization is now formed and the machinery established for carrying on the work, and to decide, some time before the next meeting (December, 1911), whether it believes that work of the kind that the Federation has shown that it can do is, in the light of local needs, worthy of the continued loyal support of the association.

In other words, that each association decide whether it wishes to continue to support the Federation.

3. That in case an association decides that it is not wise to continue the Federation, that association be urged not to withdraw until all associations have reported on this matter and all have been notified how the vote stands and what the general feeling among the local associations is.

Report of the Committee on College Entrance Requirements in Mathematics and Science.

For your committee, appointed in June, 1909, to outline a policy on the part of the Federation toward present entrance requirements in mathematics and science, the undersigned begs leave to submit the following report, with this word of explanation:

Inasmuch as an actual meeting of your committee has proven impossible, the outline of this report was submitted to the several members for criticism and suggestions, and advantage has been taken of these suggestions in making the final form accord as nearly as possible with the general sentiment of the committee. This renders the chairman alone responsible for the final form, and the report is therefore submitted for action, with this explanation.

Your committee has conducted much correspondence with secondary teachers in all parts of the country, and finds an ever increasing dissatisfaction with the present entrance conditions. This feeling has been well expressed in a pamphlet published by the High School Teachers' Association of the City of New York, entitled, "Articulation of High School and College; or, A Plea for the Reorganization of Secondary Education." In this pamphlet there is set forth in ample detail the rapidly spreading conviction that our public high schools are seriously hampered by present college entrance requirements. Copies of this pamphlet can be secured by applying to C. D. Kingsley, 400 Fourth Street, Brooklyn, N. Y.

In the West, school and college are coöperating with marked success in bringing about a better adjustment between them.

To aid in securing an equally happy condition in the East, the Association appointed a committee of five, to whose careful study of present conditions is due the admirable presentation of facts, of practice, and of opinion contained in their pamphlet, which so well deserves full and thoughtful consideration from both school and college.

Further voice has recently been given to the same sentiment by the action of three important departments of the National Education Association. The departments of secondary education, of manual training and art, and of business education united at the last meeting in requesting the colleges to give to the matter of entrance requirements their most careful consideration.

In the judgment of your committee, the time is ripe for a formal submission of this whole subject to the various affiliated societies now allied to the Federation, so that next year we may proceed to action with a full expression of opinion from them, and thus proper and necessary action may be taken.

Your committee has therefore drafted resolutions hereto attached, formulating their ideas of the needed changes in the matter of entrance requirements, and now recommends that these resolutions be referred to the various affiliated societies for their consideration and action during the coming year.

Respectfully submitted for the committee.

WILLIAM M. BUTLER (Yeatman High School, St. Louis, Mo.), *Chairman.*

Resolutions Submitted for Adoption.

Whereas—The recent extraordinary economic progress in the scientific and industrial worlds has created new and profound educational problems which can only be solved by careful investigation and experimentation, conducted by men experienced in the work of these schools, and with the aid and coöperations of the college, as well as of the business community, and

Whereas — The present high school courses have been subjected to trenchant criticism, especially from the industrial and business worlds, chiefly because present courses give insufficient attention to vocational training or to the future work of the child, and this has been one of the causes contributing to the loss from the high school of both boys and girls who would profit largely by courses that would more directly prepare them to meet the actual demands of business and of manufacturing life; and

Whereas—Although we recognize the great benefits that have come in the past to the secondary school through college entrance requirements, we yet believe that the present excessive severity of these requirements along certain traditional lines and the failure of the colleges to recognize the educational value of vocational courses toward college admission, are conditions which very seriously hamper the freedom of the secondary schools and prevent necessary investigation, repeated experiment, and successful development of courses to meet present needs and educational growth; and

Whereas—The present "unit system" of admission to college, after sufficient trial, has proven unsatisfactory because of the difficulty of reaching fair and suitable definition of these units and of properly crediting them, thus provoking endless discussion between school and college, as well as causing never ceasing friction in administration; and

Whereas—The colleges have long been urging the secondary schools to employ none but college graduates as teachers, and the schools have so far as the college offered trained teachers endeavored to comply with this request: therefore be it

Resolved—That we urge the colleges to abandon the "unit system" and in its place to accept the certificate of the high school at its face value for such work as it covers, and permit this to entitle the student to take up such college work as his preparation may warrant, whenever the certificate stands for four years of systematic and thorough training in a good high school; and

Resolved—That we recommend that the merits of the high school for such certification be determined by conference between schools and colleges in such associations as the North Central Association of Secondary Schools and Colleges, so that due weight may be given both to what the colleges desire and to what the schools can safely undertake; and

Resolved—That we request the colleges to consider whether the work done by its students in college does not in large part furnish a better basis for testing the efficiency of school preparation than do the present methods of entrance examination and of official inspection; and

Resolved—That as we consider the larger and the more important duty of the secondary school is the preparation of the students for immediate entrance upon useful life in their own communities, we believe the college should cease to discriminate against subjects that the schools find necessary in preparing their pupils for such duties; and

Resolved—That we invite the college to come into more intimate contact with the secondary schools by requiring their professors who give general courses to do some visiting of the secondary schools at frequent intervals, so that they may acquire a better personal acquaintance with high school work and high school conditions; and

Resolved—That we urge upon college men that they take a larger part in the work of teachers' associations, so as to secure real acquaintance and earnest coöperation, based upon mutual consideration and esteem; and

Resolved—That we urge the colleges to offer greater facilities for the adequate training of teachers; that liberal courses in pedagogy be established and ample opportunities be offered for practice teaching. Such influences for professional training, we believe, will do far more to raise the standard of work in our high schools than the present entrance requirements or system of inspection.

Report of the Committee on Logarithmic Tables.

Your committee to consider the question of the best form of logarithmic tables for use in college entrance examinations begs leave to submit the following report.

The committee began its work on the assumption that the principal question to be decided was whether on the whole four-place tables or five-place tables will best serve the purpose stated, and that tables of less than four places of decimals or more than five places are used so seldom in school work as not to require serious consideration in this connection. As a result of extensive correspondence with teachers both in colleges and secondary schools, the committee found that this assumption was fully justified. When it came to a choice between four-place and five-place tables, however, the committee found both sentiment and practice pretty evenly divided. Amongst the larger and more influential colleges and universities in the East and Middle West, both sides of the question are represented, with predominance, so far as numbers are concerned, in favor of five-place tables. It is to be noted, however, that with most of the colleges which admit students only by examination, the practice is to use four-place tables in the entrance examinations, and the sentiment in favor of this practice is with them very pronounced.

The secondary schools are much more strongly on the side of five-place tables. This is especially true of the public high schools, including the large city high schools, although there are exceptions. The private schools, whose business is predominatingly college preparation, are again pretty evenly divided. Some reported to the committee that they prefer and use five-place tables in their work, even though it involves the necessity of giving many of their pupils special training in the handling of four-place tables for the college examination. With many if not most of these schools, however, the desirability of uniformity of practice in the college examination outweighs other considerations.

On the whole the committee concludes that the sentiment in favor of the use of four-place tables in college entrance examinations is growing, though it has not yet by any means reached such a measure of predominance as would entitle it exclusive recognition.

Three subordinate questions came up for the committee's consideration. These were:

(1) Whether or not the tables used in examination should be provided with tables of differences and proportional parts.

(2) Whether with four-place tables they should be arranged for the sexagesimal or decimal division of the degree.

(3) Whether negative characteristics, when they occur, should be printed as such, or increased by 10.

With regard to (1) the committee is of the opinion that the tables provided at college entrance examinations should not be furnished with

tables of differences and proportional parts. In reaching this decision the committee considers that many pupils use in school tables which are not provided with these helps, and that where this is not the case they will undoubtedly have had such training as would enable them to dispense with their use. Moreover, it is more confusing to a student unfamiliar with these auxiliary tables to have presented to him at an examination a table which is provided with them, than would be the case where the conditions are reversed.

With regard to (2) the committee has found that the greater number of those who favor the use of four-place tables prefer also to have them arranged for the decimal division of the degree.

Upon (3) the committee found that sentiment is generally in favor of the prevailing practice of printing the characteristics 9, 8, etc., in the trigonometric tables instead of the corresponding negative characteristics.

For the reasons set forth above the committee makes the following recommendations, viz.:

I. That in college entrance examinations involving logarithmic computation, both five-place and four-place tables be provided, and the student be given the option which he will use.

II. That with five-place tables the trigonometric functions be given for every minute of the quadrant, but that with four-place tables they be given for every tenth of a degree, except that for the first eight or ten degrees they should be given for every hundredth of a degree; that no tables of differences and proportional parts be provided; and that logarithms with negative characteristics have their characteristics increased by 10.

The committee begs that the Federation will accept this report and discharge them from further consideration of the subject.

FLETCHER DURELL,
VIRGIL SNYDER,
JOHN H. DENBIGH,
WILLIAM B. CARPENTER,
EDWIN S. CRAWLEY, *Chairman*.

Report of the Committee on Bibliography.

Your Committee on Bibliography of Science Teaching begs to report that the Bibliography has been accepted for printing by the United States Bureau of Education. It is now in type, the proof has been read, and printed copies should be available at a very early date. R. E. DODGE, *Chairman*.

Report of the Committee on Geometry Syllabus.

The committee of fifteen on Geometry Syllabus, which was appointed at the Baltimore meeting, has been working in three subdivisions of five members each under the chairmanship respectively of Professor D. E. Smith of Columbia University, Professor E. R. Hedrich of the University of Missouri, and Professor H. L. Rietz of the University of Illinois. The first division has under consideration definitions and logic, the second minimum and full list of theorems, and the third problems and applications. The work of each subcommittee has been fully developed and transmitted to all members of the whole committee for further criticisms and suggestions, previous to a meeting of the subchairmen and several other members held at Cleveland on November 25 and 26, 1910, at which time the whole matter was worked over item by item and substantial agreement was reached on all points.

The plan is now to print a preliminary edition of two or three hundred copies to distribute among a selected list of teachers throughout the country for still further suggestions and criticisms previous to the presentation of a further report at the meeting of the National Education Association in July, 1911, and the final report to the Federation in December, 1911. (The report is being printed in this Journal, the first third appearing in this issue.) It is believed that funds can be secured for printing the preliminary edition of the report, but it is hoped that the Federation will be able to furnish the funds for its final distribution to practically all teachers of geometry in the country.

Respectfully submitted,

H. E. SLAUGHT, *Chairman.*

Report of the Committee on a Mathematical Journal.

The chairman of your committee on a mathematical journal has felt that the teachers of mathematics of this country should have a chance to say whether or not they desired a magazine devoted entirely to the interests of the teaching of mathematics. In order to get at this and other related questions, a questionnaire was sent to all members of our committee. Replies have been received from most members of the committee, but not, as yet, from all. For the most part those who did reply gave their opinions as to the feeling of the members of their associations without having the question presented and a census of the views of the members taken. This was due to the fact that the replies were sent before the various associations had held meetings. Your committee, therefore, has no complete report to make at this time. From the replies received, however, it does not seem to the chairman very likely that the committee will be able to come to any definite agreement, and if the Federation thinks best it may perhaps be discharged.

W. H. METZLER, *Chairman.*

Report of Committee on the Chemistry Unit.

The special object for the existence of this committee is to effect a full and satisfactory correlation between secondary school preparation and college entrance requirements in chemistry.

The boundaries of chemical knowledge are widening with alarming rapidity. The high school teacher is becoming bewildered at the prospect. Theoretical, descriptive, analytical, organic, physical chemistry all clamor for treatment, and the college examiner is prone to echo this demand in his requirements. A liberal use of the pruning knife is pressingly needed; only essentials must be left; unless we are willing to sacrifice thoroughness to superficiality.

The statement of the requirement in chemistry issued by the College Entrance Examination Board and endorsed by many leading Eastern colleges and universities, requires four pages to express its demands upon the high school teacher for a one year course in chemistry. The actual examinations set by the examiners of this same board are, however, exceedingly reasonable. It is high time, then, that the discrepancy between anticipation and realization be minimized. Prompt action on the part of those in charge of these requirements will meet without question the heartiest approbation of chemistry teachers throughout the East.

The committee will feel grateful for suggestions or information from any source pertinent to work in its appointed field.

Respectfully submitted,

HAROLD BISBEE, *Chairman.*

SCIENCE QUESTIONS.

By Franklin T. Jones,

University School, Cleveland, Ohio.

Our readers are invited to propose questions for solution—scientific or pedagogical—and to answer the questions proposed by others or by themselves. Kindly address all communications to Franklin T. Jones, University School, Cleveland, Ohio.

Questions and Problems for Solution.

45. *Proposed by Chas. H. Korns, Bradford, Pa.*

A team of horses pulls a wagon and load up a hill. To hold the load on the hill requires a force of 4,000 pounds. Neglecting friction, do they pull more or less than 4,000 pounds, or just 4,000 pounds, in pulling it up at a uniform rate? At an accelerated rate? Do they pull any harder than the load does?

Note.—Compare this problem with No. 34. Which answer was correct—the one which took account of acceleration or the one which did not?

46. *Proposed by O. R. Sheldon, Chicago, Ill.*

Why can one skate on "rubber ice" when the ice would break if he stood still?

47. *Proposed by Roy E. Bowman, Alliance, Ohio.*

A wire weighing 0.375 pounds per foot is supported by two posts 160 feet apart. The horizontal pull on the wire is 400 pounds. Find the deflection at the center and the length of the wire.

Note.—Mr. Bowman "would like to see a simplified exposition of hyperbolic functions given in the Science Notes" and "to bring it about proposes the above problem."

Sheffield Scientific School Entrance Examination.
June, 1910.

1. Write equations for the reaction between silver and nitric acid; also for the reaction of the resulting solution with hydrochloric acid. Describe the visible changes accompanying these reactions.

2. Write equations showing the reactions of dilute sulphuric acid with the following: (1) Sodium sulphide, (2) potassium carbonate, (3) ammonium hydroxide, (4) cupric oxide, (5) aluminum hydroxide, (6) lead nitrate, (7) barium chloride, (8) zinc.

3. State Boyle's law and Charles' law. A certain quantity of a gas occupies one liter at O°C and 760 mm. Calculate its volume at 20° C and 750 mm.

4. Give the formulas for two common oxides of phosphorus; also for two common oxides of arsenic. Write equations for reactions showing the formation of two salts containing phosphorus and of two salts containing arsenic.

5. Calculate the weight of chlorine that could be produced by the action of manganese dioxide on 50 g. of hydrochloric acid.

Solutions.

36. *Proposed by John C. Pickard, Brookline, Mass.*

Wooden sphere, specific gravity .7. To what depth will it sink in salt water, specific gravity 1.026?

Solution by Roy E. Bowman, Alliance, Ohio.

Density of wood=.7

Sp. G. of salt water=1.026

$\therefore$ 1 cu. ft. wood displaces $\frac{.7}{1.026}$ cu. ft. salt water.

Let x=radius of ball

y=height of segment of ball immersed in salt water

(1) $\frac{4\pi x^3}{3}=1$, vol. of ball

(2) $\frac{\pi x^2 y}{2}+\frac{\pi y^3}{6}=\frac{.7}{1.026}$ =Part of ball submerged

(3) $x=\sqrt[3]{\frac{3}{4\pi}}=.62034$

(4) $\frac{\pi y^3}{6}+y\left(\frac{\pi}{2}\sqrt[3]{\frac{9}{16\pi^2}}\right)=\frac{.7}{1.026}$ (Substituting value of x).

(5) $y^3+1.1545y-1.3030=0$ (Simplifying, using 5 place logs).

(6) By trial, $y=.7555$

(7) Ball sinks $\frac{.7555}{1.24068}$ or .6089 of its diameter

(8) $\therefore$ 60.89% of ball's diameter sinks in the salt water.

Also solved by H. A. Porter, Liberty Hill, Texas.

37. *From an entrance examination paper of Sheffield Scientific School.*

A boy throws a ball vertically upward, and four seconds later it strikes the earth. What height did the ball attain, and with what velocity did it leave the boy's hand?

Solution by C. B. Brown, Madisonville, Tenn.

The total height reached by the missile is equal to the product of one half the acceleration times the square of the time. Time going up is half whole time till return, or 2 sec.; i. e., $S = \frac{1}{2}at^2$ $G = a = 32.2$ ft. per sec.^{2}

$$= \tfrac{1}{2} \times 32.2 \times 2^2$$
$$= 64.4 \text{ ft.}$$

The velocity (initial) is the same as the final velocity in such cases as this, or, is equal to the acceleration $\times$ the time, $V = at$

$$= 32.2 \times 2$$
$$= 64.4 \text{ ft. per sec.}$$

Also solved by Wm. E. Wing, H. A. Porter, C. A. Perrigo.

38. A dog hitched to a cart can exert a pull of 10 kg. With what velocity must the dog move to do work at the rate of 20 kilogrammeters per second?

Solution by H. A. Porter, Liberty Hill, Tex.

10 kg. = 10,000 g.; 1 g = 980 dynes.

10,000 g $\times$ 980 = 9,800,000 dynes = force the dog exerts.

9,800,000 dynes acting over 1 cm. = 9,800,000 ergs = work he does in going 1 cm.

1 kilogrammeter = 98,000,000 ergs.

20 kilogrammeters = 20 $\times$ 98,000,000 ergs =1,960,000,000 ergs = amount of work to be done in 1 second.

$$\frac{\text{1,960,000,000 ergs, amount of work to be done in 1 second}}{\text{9,800,000 ergs, amount he does in going 1 cm.}} = \text{200 cm. the veloc. per sec.}$$

200 cm. = 2 meters per second, velocity of dog.

NOTE BY EDITOR—Since we wish to know how far per second (velocity) the dog will move and since work = force × distance,

20 kg.—m. = 10 kg. × distance, or distance = 2 m. in one second.

Also solved by Wm. E. Wing, C. B. Brown.

39. A twenty gram weight attached to one end of a uniform rod 100 cm. long causes it to balance about a point 20 cm. from that end. Find the weight of the rod.

Solution by C. A. Perrigo, Dodge, Neb.

Considering gravity applied at middle point of rod and remembering moments are equal in each case we have

$$30x = 20 \times 20 = 400$$
$$\therefore x = 13\tfrac{1}{3} \text{ grams.}$$

Also solved by H. A. Porter, W. E. Wing, C. B. Brown.

40. A rectangular block of stone, 20 × 30 × 40 cm., has a density of 2.0 gm./cm.³ What force will be required to lift this block under water?

Solution by Wm. E. Wing, Deering High School, Portland, Me.

Volume of block = 20

$$\begin{array}{r} 20 \\ \underline{30} \\ 600 \\ \underline{40} \\ 24{,}000 \text{ c. c.} \end{array}$$

Wt. of block 2 × 24,000 = 48,000 gms.

Volume of water displaced = 24,000 gms.

Buoyancy of water = 24,000 gms.

48,000 — 24,000 = 24,000 gms. force required.

Also solved by C. A. Perrigo, H. A. Porter.

41. What must be the length of an open organ pipe that will give a fundamental note of 200 vibrations per second, if the velocity of sound in air is 300 m./sec.?

Solution by H. A. Porter, Liberty Hill, Tex.

The wave length $= \dfrac{300\,m}{200} = \tfrac{3}{2}$ meter, wave length of fundamental note.

In open organ pipe the length of pipe = $\tfrac{1}{2}$ wave length of fundamental note; therefore length of pipe

$$= \tfrac{1}{2} \times \tfrac{3}{2}\, m. = \tfrac{3}{4} \text{ meter} = 29.5275 + \text{inches.}$$

Also solved by C. B. Brown, C. A. Perrigo.

42. A piece of copper weighing 200 gm. is taken from an oven and placed in 500 c.c. of water. The temperature of the water is changed from 20° C. to 30° C. If the specific heat of the copper is 0.69, what was the temperature of the oven?

[NOTE—The author of this question evidently failed to consult a table of specific heats.—ED.]

Solution by C. A. Perrigo, Dodge, Neb.

Let x = temperature of oven.

Now M. T. S. of copper = M. T. S. of water.

Substituting $200(x - 30°) \times .69 = 500 \times 10 \times 1$

$$138x - 4{,}140. = 5{,}000$$
$$\therefore x = 66.2° + \text{temperature of oven.}$$

Solved also by H. A. Porter.

ARTICLES IN CURRENT MAGAZINES.

American Forestry for February: "Present Forestry Issues," by Curtis Guild; "Shall States Regulate the Management of Private Forests?" by Herman H. Chapman; "City Trees and Their Relation to Forestry," by J. J. Levison.

Education for February: "How can the High School Serve More Effectively the Interests of the Community," Charles F. Harper; "Manual Training and Local Industry, A Course of Study for Boys of the Two Upper Grades," Ernest B. Kent; "The Home and School Life," J. M. Greenwood; "The Problem of Public Education," Henry L. Upton.

Popular Science Monthly for March: "Ehrlich's Specific Therapeutics in Relation to Scientific Method," Dr. Fielding H. Garrison; "The Disciplinary Value of Geography," Professor W. M. Davis; "The Work of the 'Albatross' in the Philippines," Albert I. Barrows; "The Story of a King and Queen," Professor Cyril G. Hopkins; "The Social Problem," Professor John J. Stevenson; "Motor Education for the Child," Dr. J. Madison Taylor; "The Case of the College Professor," Professor Warner Fite; "The Consulting Psychologist," Professor C. E. Seashore; "The Work of Chemistry in Conservation," Professor Elbert W. Rockwood.

Popular Astronomy for March: "Variable Star Work for the Amateur with a Small Telescope," William Tyler Olcott; "Interesting Cometary Statistics," H. C. Wilson; "Celestial Photography," Arthur K. Bartlett; "Power from the Sun," Alfred Rordame; "Seeing Stars in the Daytime," Harold B. Curtis; "Foucault's Pendulum Experiment," Edison Pettit; "The Nature of Coronium in Sun and Star," Monroe B. Snyder; "The Total Solar Eclipse of 1911 April 28," Pio Emanuelli.

Physical Review for February: "Measurements of the Rate of Decay of Gas Phosphorescence," C. C. Trowbridge; "Experiments in Impact Excitation. III. The Frequency of the Lepel Oscillations," George W. Nasmyth; "The Hall Effect and Some Allied Effects in Alloys," Alpheus W. Smith; "On the Secondary Relation from Solids and Liquids," S. J. Allen; "A Simple Method for Determining Nocturnal Radiation Proposed by K. Angström," A. K;Son Angström; "Formation of Ozone," A. W. Ewell; "The Kinetic Pressure-Drop Correction in the Transpiration Method for Gas-Viscosity," Willard J. Fisher.

School Review for February: "Entrance Requirements and 'College Domination' as Sources of Motivation in High School Work," William C. Bagley; "The Relation of the Movement for Vocational and Industrial Training to the Secondary Schools," Frank M. Leavitt; "What the University Expects of High School Students in English," James Weber Linn; "A Visit to the Frankfort *Musterschule*," Charles Goettsch; "Some New Modifications of Old Experiments in Physics," E. S. Bishop; "The First Year Science Course in High School," Wallace W. Atwood.

Scientific American for February 25: "New Things in Aëronautics;" "Labor-Saving by Automobile Power;" "Observations Among the Workshops of Europe;" "Fighting Death in Mines."

Zeitschrift für den Physikalischen und Chemischen Unterricht for January: W. König, "Zwei Modelle zur Optik;" K. Noack, "Untersuchung der Ablenkung, die ein Magnet an einer Kompatznadel hervorruft, um die er in verschiedenen Stellungen im Kreise herumgeführt wird;" O. Dörge, "Schulversuche aus der Akustik;" W. Volkmann, "Ein Lichtzeiger für objektive Spiegel-ablesung;" W. Jaeckel, "Uber eine einwandfreie Ausgestaltung des Kräfteparallelogramm-Versuches im Falle zweier unter rechtem Winkel angreifender Komponenten."

Journal of Geography for February: "An Orthodox Aspect of Predestinarianism," Jacques W. Redway; "Commercial Geography in Philadelphia," Wesley N. Clifford; "The Point of View in Elementary Geography," J. Paul Goode; Symposium on Commercial Geography.

Nature-Study Review for February: "Shade Tree Protectors' League of Newark, N. J.," Agnes V. Luther; "Human Interest in Trees," Ruth Marshall; "Pupil's Observation of an Ant's Nest," Grace J. Baird; "The Oaks," Fred L. Charles; "The Ginkgo," Emilie Yunker.

Mining Science for February 9: "Gold Ore Crushing System in Russia," by H. C. Bayldon; "The Nevada City Mining District," by Al. H. Martin. For February 23: "Electric Equipment of California Dredges Principles and Applications of Electrostatic Separation," by F. S. Macgregor.

School World for February: "Curriculum of Training Colleges"; "Education in Citizenship," by Miss Geraldine E. Hodgson; "The Teaching of Geography in Secondary Schools" (illustrated), by B. C. Wallis; "Day Training Colleges in England," by Peter Sandiford; "Some Experiments in the Teaching of History," by Arnold Smith.

SCIENTIFIC STUDY OF EDUCATION IN BIOLOGY—BIBLIOGRAPHY.

Under this caption there will be published from time to time a list of books and papers believed to be of interest to those interested in the subject. This is done as a part of the work of the committee appointed by the Biology Section of the Central Association for the purpose of encouraging research in the pedagogy of biology.

Three classes of titles will be included as follows: (1) text-books and other works of general interest, but bearing upon the subject, (2) investigations the subject matter of which is not biological but which are of interest to biologists because the method may be applicable in biology and, (3) investigations in the pedagogy of biology. It is hoped to make the third division complete; in the other two only the most important titles can be included.

It is hoped that members of the section, authors of papers, and publishers will coöperate by calling to the attention of the committee such papers as might not otherwise be noted.

Caldwell, O. W.
An Investigation of the Teaching of Biological Subjects in Secondary Schools. SCHOOL SCIENCE AND MATHEMATICS. June, 1909.

The questionnaire method is used in securing the data upon which the study is made. A valuable paper.

Hunter, G. W.
The Methods, Content, and Purpose of Biological Science in the Secondary Schools of the United States. SCHOOL SCIENCE AND MATHEMATICS. January and February, 1910.

A very extended investigation by questionnaire of the actual conditions in the field indicated in the title. On some points the only available source of definite information.

Caldwell, O. W.
Botany in the Schools, Etc. Cyclopædia of Education. Article on Botany. Vol. 1. N. Y., 1911. Macmillan and Company.

The latest and most authoritative summary of present conditions.

Lucas, F. C.
Two Experiments in Zoölogy Teaching. SCHOOL SCIENCE AND MATHEMATICS. *11*:107, 108. February, 1911.

A very brief presentation of results secured in segregated classes, and by certain variations in the material of the course. The brevity of the paper unfortunately compels omission of detailed data.

Gartland, Peter G.
Two Experiments on Grammar School Graduates. SCHOOL SCIENCE AND MATHEMATICS. *11*:155-159. February, 1911.

An analysis of results form two sets of examination papers of the usual sort. Particular reference to percentage of error and inefficiency. An illustration of amount of unused material in every examination.

Ganong, W. F.
The Teaching Botanist. xi+439. New York: 1910. Macmillan.

This is not a book upon experimental pedagogy, but it is a most inspiring and useful discussion of the teacher, his tools and his work. It is the standard authority and should be within the reach of every teacher of botany.

Stark, Wm. E., and others.
The Teaching of Mathematics in the Private Secondary Schools of the United States. School Science and Mathematics. *11*:133-154, 225-241. February and March, 1911.
An exhaustive examination of a clearly limited field by a committee reporting to the International Commission. Of interest to biologists as suggesting the sort of careful and comprehensive investigation that ought to be made in biology.

ACTION OF ACID FUMES AND ALKALI UPON CONCRETE WORK.

One of the most useful and important all round building materials which has come into such universal demand comparatively recently is Portland cement concrete. It is manufactured in such enormous quantities both in the United States and Canada from materials which are in themselves not expensive that it is now being substituted in hundreds of ways for the more expensive wood, brick, and in many cases stone work. The ease with which it can be moulded into any desired shape renders it a valuable building material. What perhaps is its most important qualification is its unquestioned durability when exposed *only* to the action of fresh air and water. But when subjected to the action of alkali waters or acid fumes it rapidly disintegrates and becomes worthless for the purpose for which it was intended. Many instances have recently been reported of the deterioration of concrete work in the neighborhood of smelters, mines, and reduction works where quantities of acid fumes are belched forth from the stacks. A notable example of where alkali waters have acted upon Portland concrete to its disintegration is in the Tansil dam of the Pecos River in New Mexico. In this case large cavities were produced by the action of the alkali water on the lime in the concrete. Frequently sea water will attack concrete work and by chemical action render it worthless in a comparatively short time.

It is on account of the large percentage of CaO in the composition of Portland cements which render them so liable to chemical decomposition. Doubtless this high ratio of lime will give to the cement a high degree of strength in short time tests, but in an endurance test when acid fumes, alkali, or even sea water may come in contact with the cement then it is that this rapid chemical decomposition takes place. A Portland which is high in its percentage of SiO_2 and comparatively low in CaO is one which is not chemically acted upon by the conditions named. A cement of this type is now being made which not only possesses all the good qualities of high CaO cements but is at the same time not acted upon by the agencies named. Tests show that this new Portland possesses a strength much greater than that of the high lime cements. It can be used under any condition and as far as known will not disintegrate. Its specific gravity is 2.95, while that of the regular cement is 3.10. This is another element in its favor.

BIBLIOGRAPHY ON MATHEMATICAL WORKS.

Teachers College, Columbia University, New York City, has recently issued a bibliography of mathematical works suitable for high school and normal school libraries. This is prepared by Professor David Eugene Smith and Professor Clifford Brewster Upton. Teachers who may care for this bibliography may obtain it gratis by writing to the Secretary of Teachers College, Columbia University.

BRASS COATING ON COPPER WIRE.

A brass coating may be put upon copper wire by introducing the wire into a retort in which raw zinc has been heated to volatilization. In the presence of the copper the retort is heated to a temperature of 900 to 1,020 degrees C., such as to affect the outer portion of the copper. The zinc vapor combines with the heated copper and the latter is allowed to remain in the retort until the brass coating has the desired thickness. During a subsequent drawing process the coating follows the wire, and the luster is secured by burnishing or polishing during or after the drawing process. The degree of fineness to which such a wire is to be drawn determines the amount of brass coating applied.

REPORT OF THE NEW ENGLAND ASSOCIATION OF CHEMISTRY TEACHERS.

The fortieth meeting of the Association was held Saturday, January 28, 1911, at the Classical and High School, Salem, Massachusetts, following a trip through the tanneries of the National Calfskin Company at Peabody, Mass. The meeting was called to order by the president, Mr. Frederick C. Adams. After the election of new members and the reports of standing committees, President Adams announced the recent incorporation of the Association and the appointment of a Committee on Library and Museum with Professor Lyman C. Newell as Curator. This committee is to begin at once the collection of all kinds of material that will be of service to the members of the Association engaged in teaching. Prof. Newell spoke briefly of what the Association aims to do for its members by the maintenance of a library and museum.

The opening address was by Mr. Alan A. Claflin, president of the Avery Chemical Company, Boston, who spoke on "The Manufacture of Leather from the Point of View of the Chemist." After mentioning briefly the earlier methods of tanning, he outlined the history of the chrome tanning process, and discussed the principles involved in this new method, which in less than twenty years has revolutionized the leather industry. The closing address was by Mr. William Armstrong, President of the Armstrong Leather Company of Peabody, who spoke on "The Raw Material and the Finished Product," illustrating his most entertaining remarks with many handsome specimens of leather.

The meeting adjourned with a vote of thanks to the speakers of the day and to Mr. Francis R. Hathaway, head of the science department of the Salem High School, who arranged the details of the meeting, including the trip through the tannery and the excellent luncheon that preceded the afternoon session. EDWARD S. BRYANT, Secretary.

IOWA ASSOCIATION OF SCIENCE TEACHERS.

The Iowa Association of Science Teachers held its annual meeting in Des Moines in connection with the Iowa State Teachers' Association on November 4 and 5, 1910.

The section meetings which were held separately this year for the first time were unusually well attended. The biology section led by Professor R. B. Wylie of the state university dealt exclusively with the elementary course in botany. The apparatus for the course, the semester course, and the possibility of the Iowa schools meeting the requirements of the new unit recommended by the North Central Association were discussed by Mr. Wilson of

Upper Iowa University, Miss McFarland of West Des Moines, and Mr. Ewers of Davenport.

The chemistry section discussed informally the report of the committee on correlation of high school and college chemistry. The committee was unable to make definite recommendations and was continued. Professor A. A. Bennett of Ames is chairman of the committee.

The physiography section led by Miss Frances Church of East Des Moines took up the humanizing of secondary school geography. The sentiment seemed to be that not much should be attempted in the way of application save as sidelights thrown by the teacher into the discussion of the work of the class. Professor E. J. Cable of the state teachers' college presented a list of fundamental exercises in physiography as used in the college and suggested their adaptation to secondary schools. A committee was appointed to report on work which should be included in a high school course in physical geography.

The physics section led by Professor D. W. Morehouse of Drake University considered the following topics:

"The 'Pros' and 'Cons' of Cheap Laboratory Apparatus," Lafayette Higgins, West Des Moines.

"The Physics of Toys," W. H. Gemill, Dallas Center.

"The Value of Illustrations and Applications in the Teaching of Physics," Lewis B. Mull, Ottumwa.

"A 'First Course in Physics' in College," F. F. Almy, Iowa College.

There was a strong feeling expressed by those present that the colleges ought to give more recognition to the work in physics done in the high school.

The general session of the Association was held on Friday, November 5th. Professor R. K. Duncan of the University of Pittsburg gave an address on "Some Problems in Industrial Chemistry."

The following officers were chosen for the ensuing year:

President, Professor R. B. Wylie, State University of Iowa.

Vice-President, Emma J. Fordyce, Cedar Rapids.

Secretary-Treasurer, F. E. Goodell, North Des Moines High School.

Section Leaders—Physics, Lewis B. Mull, Ottumwa High School; Chemistry, J. R. Townsend, Fort Dodge High School; Physiography, Maud Brown, West Des Moines High School; Biology, A. F. Ewers, Davenport High School.

ASSOCIATION OF OHIO TEACHERS OF MATHEMATICS AND SCIENCE.

The Association of Ohio Teachers of Mathematics and Science held its eighth annual meeting in Chemical Hall, Ohio State University, Columbus, Ohio, December 28, 29, 1910.

The first number on the program was a paper on "Earth Science" by William P. Holt of Central High School, Toledo, Ohio. Mr. Holt first took up a defense of the earth science course in high schools. He said that most of the criticism of this course was due to the meager and aimless work attempted by many schools. He showed how it could be made more useful by a study of bed rock, soils, etc., and by collections and indoor field work by means of pictures, lantern slides, etc.

The Association then adjourned to sectional meetings.

The first work of the mathematics section was a paper on "Mathematics" by Mrs. Eva S. Maglott of Ohio Northern University, Ada. She gave a general discussion of the field of mathematics from the simplest attempts to understand nature, to the mathematics of the present day, which is the

basis of modern achievement. She contended that mathematics is an independent science and must develop in its own domain; that it possesses an ideal of exactitude—the attainment of perfection—which gives to the student a new standard of rigor in work which extends beyond the subject to methods of teaching and studying.

"Sequence in Algebra," was the subject of a paper by O. A. Bailey of the high school of Piqua, Ohio. In his paper, Mr. Bailey said that it was absolutely necessary at the outset to have a correct understanding of algebraic symbols. Factoring depends on a correct understanding of the expression factored; fractions and fractional equations should be studied by application of axiomatic principles, while the general solution of the quadratic depends on the condition of the perfect square. He said mathematics was causing boys and girls to discontinue school and suggested as a remedy that the work be made more interesting, and that there should be a higher standard of teaching and better plans for the work. This paper was followed by vigorous discussion of the topics it treated.

A heated discussion prolonged far beyond the dinner hour was aroused by the address of T. M. Smith, Lash High School, Zanesville, Ohio, who speaking on the subject, "The Definition in Geometry," pointed out the evils of loosely defined fundamental concepts in geometry, and vigorously condemned the slipshod methods of the average teacher in not demanding better things of text makers and in not using more practical and intensive methods of instruction.

The inordinate demand for "sugar-coated" texts is beside the point. If the teacher is a "live wire," the text matters little.

Best of all, the speaker illustrated his views by a life-sized cross section of a geometry recitation, in which even the dullest boy and thoughtless miss could not fail of enjoyment and profit.

The work of the science section was opened by a paper on "The Textbook," by Fred J. Hillig, professor of physics, St. John's College, Toledo, Ohio.

He thinks that most of our text-books on physics attempt to give too much detail and explanation. The purpose of the text-book is not to teach but to serve as a guide to both student and teacher. The book should not contain the whole of the explanation but the substance of the facts that are to be studied.

Neither should it contain more cuts than are needed to make the meaning clear. The book that attracts too much, detracts from the teacher.

"Methods of Teaching Physics," was the subject of a paper by John Whitmore, professor of physics, University of Wooster, Wooster, Ohio.

He said that physics should be taught from a threefold viewpoint: namely; of intellect and instruction, of truth and veracity, and of teaching the pupil to depend on his own judgment. He considered the kinds of courses offered, amount and purpose of laboratory work, etc. He also considered the difficulties in the way of good physics teaching. Some of these are lack of preparation and fitness on the part of the teacher and the preparation and previous instruction of the pupil.

Several pieces of simple but efficient apparatus were shown and explained, among which were the hydrostatic balance by Professor Fred J. Hillig, an instrument for determining index of refraction by Professor J. A. Culler, Boyle's Law apparatus and an arrangement for demonstrating the difference between electromotive force and potential difference by Mr. W. C. Phebus.

Professor J. L. Gilpatrick gave a partial report for the "Committee on Arrangement of the Mathematics Curriculum." He reported that not much progress had been made. Professor K. D. Swartzel of Ohio State University, Principal D. C. Rybolt of Akron, and Principal D. R. Ellabarger of Piqua

were retained on the committee and instructed to continue the work. And it was suggested that a large part of the next program deal with this subject.

The question of a time of meeting that would be more favorable was freely discussed and a majority of those present favored a change of time.

The following officers were elected for the ensuing year:

President, William P. Holt, Central High School, Toledo.

Vice-President, Ralph W. Buck, Stivers Manual Training School, Dayton.

Secretary-Treasurer, Miss Harriet E. Glazier, Western College for Women, Oxford.

Assistant Secretary, R. O. Austin, School of Commerce, Columbus.

CHARLES T. PROSE, Secretary.

MINNEAPOLIS MEETING OF THE AMERICAN NATURE-STUDY SOCIETY.

The fourth annual meeting of the American Nature-Study Society, held Friday, December 30, 1910, attracted a considerable number of members from several different states. The morning session was devoted to the general topic, "The School Garden as a Nature-Study Laboratory." Garden work in Minneapolis was presented by Mary D. La Rue, principal of the Pierce School, who gave an interesting account of work done under her direction. She exhibited views of the children's gardens and displayed pupils' hand-work in nature-study, art, and language. She was followed by Mr. Leroy J. Boughner, editor of the Minneapolis *Tribune*, who gave the experience of that paper in encouraging vacant lot gardens. Mr. D. Lange, principal of the Humboldt High School, St. Paul, and author of "Handbook of Nature-Study," then told of garden experiences in the St. Paul public schools, from which interesting conclusions were drawn.

"South Chicago School and Home Garden Work" was presented by Professor Otis W. Caldwell, president of the American Nature-Study Society, whose admirable stereopticon views and graphic description of backyard gardens in poor districts carried a message to all interested in childhood. J. A. Drushel of Teachers' College, St. Louis, outlined "School Garden and Greenhouse Work in St. Louis." His talk was illustrated. Several members participated in the discussion of the general topic.

At the business session the secretary-editor reported that the close of the year 1910 showed a paid-up membership exceeding 1,100 and a small balance in the treasury.

The following officers were elected:

President—B. M. Davis (Ohio).

Vice-Presidents—G. H. Trafton (N. J.); S. Coulter (Ind.); F. L. Stevens (N. C.); F. L. Holtz (N. Y.); and D. J. Crosby (D. C.).

Directors—Ruth Marshall (Ill.); E. B. Babcock (Cal.); J. Dearness (Ontario); Otis W. Caldwell (Ill.); and Anna B. Comstock (N. Y.).

The following directors hold over: L. H. Bailey (N. Y.); C. F. Hodge (Mass.); C. H. Robison (N. J.); S. C. Schmucker (Pa.); Delia Griffin (Vt.); Grant Smith (Chicago section); and J. A. Drushel (St. Louis section).

The afternoon session considered two topics. The first, "Natural History Museums in Relation to Nature-Study Instruction," was discussed by I. B. Meyers, who briefly presented "The University of Chicago, School of Education Plan," and J. A. Drushel, who outlined in detail "The St. Louis Plan," showing lantern views. Mr. Drushel's paper was a very complete statement of the organization and work of the St. Louis Educational Museum, an institution which, dating from the World's Fair of 1904, has developed

into one of the most prominent and useful features of the educational system of that city. A general statement of the work of this museum appeared in the *Nature-Study Review* for April, 1910, and it is hoped that Mr. Drushel's paper may be published in full in an early issue of the *Review*.

The second topic, "The Organization of Nature-Study," was discussed by I. B. Meyers, Fred L. Charles, and Otis W. Caldwell. Mr. Meyers read a very carefully prepared paper on "Our Present-Day Outlook into the Teaching of Nature-Study." Mr. Meyers has been devoting the year to the investigation of special problems in nature-study teaching. Mr. Charles made a plea for "The Standardization of Nature-Study," urging that the American Nature-Study Society has an important though difficult function to perform in endeavoring to define nature-study aims and principles and working toward the establishment of acceptable standards in the wide field it has to cover. The closing paper of the meeting was given by Professor Otis W. Caldwell, who spoke on "The Organization of the Course in Elementary Science for the Grades." His talk was most helpful, consisting of generalizations based upon experience and regarded as fundamental to any elementary course in the materials of nature-study. His paper will appear in a forthcoming number of the *Nature-Study Review* as the second of a series, the first of which appeared in the Course of Study Number, October, 1910.

FRED L. CHARLES.

The first of a series of articles, later to be issued in book form, by Dr. George Bruce Halsted, so well known to our readers, appears in February's *Open Court* under the striking caption, "The Prehuman Contributions to Arithmetic." To Dr. Halsted alone has been given the power to transmute the dry bones of arithmetic into a fascinating serial.

BULLETIN OF THE MASS. AGR. COLLEGE, DEPT. OF AGR., ED. CIRCULAR 10, 1910.

In August, 1910, the Third Annual Conference on Agricultural Education was held at Amherst, Mass. School and Home Gardens proved to be subjects of such extensive interest that the Massachusetts Agricultural College has brought together seven of the papers upon the above topic and has issued them as Bulletin 10 of the Department of Agricultural Education. The first and last of these papers deal respectively with the place of garden work in education, and suggestions as to how the work should be done. All the other articles are descriptions of what has already been accomplished, and because of their direct, concrete statement of actual occurrences in garden work are of much value. It is one thing to have springtime enthusiasm about garden work, and quite another to have the same kind of enthusiasm at the close of the season's work. For this reason especial value attaches to the many records of experience, to the details of practice, and the products that were secured. Several accounts that were written by pupils offer good measurements of work from their point of view.

A statement of educational value "after the fact" is of more significance than a prognostication. For example, one teacher, at the close of her report says: "The value of our garden was twofold. First, it put new life and interest into every subject in the school, and second, it enabled us to reach out through the garden to a small degree at least, to the community. Herein, I believe, lies the real value of any school garden." The entire pamphlet is valuable, and is a fairly good measure of the present stage of development of the school-garden movement.

O. W. CALDWELL.

WEBSTER'S NEW INTERNATIONAL DICTIONARY.

One of the most important and valuable books which should be in every teacher's and professional person's library is the volume mentioned above. Tc keep in touch with the times the book must be frequently consulted Many of our best teachers, who are the busiest, do not know how to use the dictionary wisely and with the least expenditure of time and labor. The publishers of this great book, G. and C. Merriam Company, Springfield, Mass., with their characteristic business ability, have prepared a little booklet entitled, "Suggestions on the Use of the Dictionary," which they will mail free to any teacher asking for it.

BOOKS RECEIVED.

The Ninth Year-Book of the National Society for the Study of Education. Part II. The Nurse in Education. The University of Chicago Press.

The Tenth Year-Book of the National Society for the Study of Education: Part I, The City School as a Community Center; Part II, The Rural School as a Community Center.

Annual Report of the Smithsonian Institution. 1909. x+751 pages. 16x23 cm. Government Printing Office, Washington.

Elementoj de la Geometrio Absoluta, Verkis Dro Cyrillo Vörös, Profesoro de L'Piaj Lernejoj Budapest.

Physical Geography for Schools, by Bernard Smith, Cambridge. viii+190 pages. 15x22 cm. $1.10, net. The Macmillan Company, New York.

Bulletin of the Minnesota Academy of Science. Pp. 319-462. The Telegraph-Herald, Dubuque.

Elements of Geology, by Eliot Blackwelder, University of Wisconsin, and Harlan H. Barrows, University of Chicago. 475 pages. 15x19 cm. American Book Company. New York and Chicago.

A Course in Qualitative Chemical Analysis, by Charles Baskerville and Lewis J. Kurtman, College of the City of New York. xi+200 pages. 15x22 cm. $1.40, net. The Macmillan Company, New York.

Essentials of Biology, Presented in Problems, by George William Hunter, DeWitt Clinton High School, New York. Pages 448. 15x21cm. American Book Company, Chicago.

BOOK REVIEWS.

Wild Birds in City Parks, by Herbert Eugene Walter and Alice Hall Walter. Fourth enlarged revision with chart and key. Price, 35 cents postpaid. Author's edition. A. C. McClurg and Co., Western Distributors. 1910.

The many who have been using the earlier editions will be pleased to know of this new and enlarged revision. The authors have endeavored to make the book more usuable to a wider range of students and to this end have added to the list given in former editions about 50 birds, making the book useful in the entire Northeastern United States west to the Mississippi.

The key has been carefully revised and simplified and a new table of distribution covering 350 birds added.

As the authors say, there is "need for a simple, compact field manual at a minimum price," suitable for use in schools as a supplementary textbook. In these days when identification is made easy by picture books, it is well to have some books without pictures that the good old-fashioned art

of identifying specimens may be practiced. There is danger in making the road to identification too easy to the beginner, so that when a real problem comes to him, as it surely will, he will not be too easily discouraged.

Professor and Mrs. Walter have done a great service to the birds with this little book. We trust it will find its way into many schools.

W. W.

Farm Friends and Farm Foes, A Text-book of Agricultural Science, by Clarence M. Weed, D.Sc., State Normal School, Lowell, Massachusetts. Pp. xi + 334. Price . 1910. D. C. Heath & Co.

There will soon be no excuse for failure to teach agriculture in the schools, and teach it in the right way, if books like the present one continue to be issued. While the book takes a narrow field it still has a very practical value for a course in agriculture. It is simply written and with each topic very practical and simple directions for observations and study are given. There are five divisions, taking up in succession, the friends and foes among the weeds, the insects, the fungi, the birds, and the mammals. Insects and fungi receive by far the greater portion of the author's attention as probably they deserve, but the birds would seem to be worthy of more than the twenty pages given to them. Trees are altogether omitted, though one can hardly imagine a better farm friend than they.

Under the right conditions this book would be suitable for a class in agriculture—and it should be found in all schools as a reference book for classes in biology. It is rather a pity that the publishers should issue it in cheap binding, so cheap that the volume used by the reviewer dropped some of its leaves. W. W.

Industrial Studies—United States. Nellie B. Allen. 287 pages, 17 chapters, and 125 illustrations. 1910. Ginn & Company. 65 cents.

A supplementary geographic reading book for the grammar grades. The book is similar but superior to various of the elementary reading books dealing with the food, clothing, and shelter of man which have been recently issued. It presents the various industries in their geographic aspect and carefully avoids the overemphasis of the technical processes. The author presents sound reasons for industrial studies in elementary schools. "Nine-tenths of the pupils in our schools will enter some form of industrial life. We are dependent for our food, clothing, and shelter on the great industries in our own and other countries. A practical knowledge of the United States can be gained largely through a study of its industries. Such topics as soil, surface, climate, and drainage become more definite to the child if studied as the underlying success of certain great industries." This reading book is very attractively written for elementary pupils and has evidently been carefully tested by use with students before being placed in its permanent form. A number of the recent supplementary books of this class have been so hastily compiled with diluted technical material and physical geography that they bring discredit to the new ideas in geography.

The first five chapters form a general geographic introduction to the industrial studies of the United States. The third chapter concerning surface and drainage is well done when judged by its adaption to the capacity of the pupils, the rational statements, and the information presented. A good political map of the United States would be a valuable addition to these introductory discussions of the position, surface, drainage, waterways and railroads.

The main treatment of the industries is confined to the twelve short chapters in which the following topics are discussed, Cotton, Sugar, Fruit, Wheat, Corn, Coal, Iron, Gold and Silver, Cattle and Beef, Sheep and Wool, Lumber and Fishing. At the close of each chapter there are well

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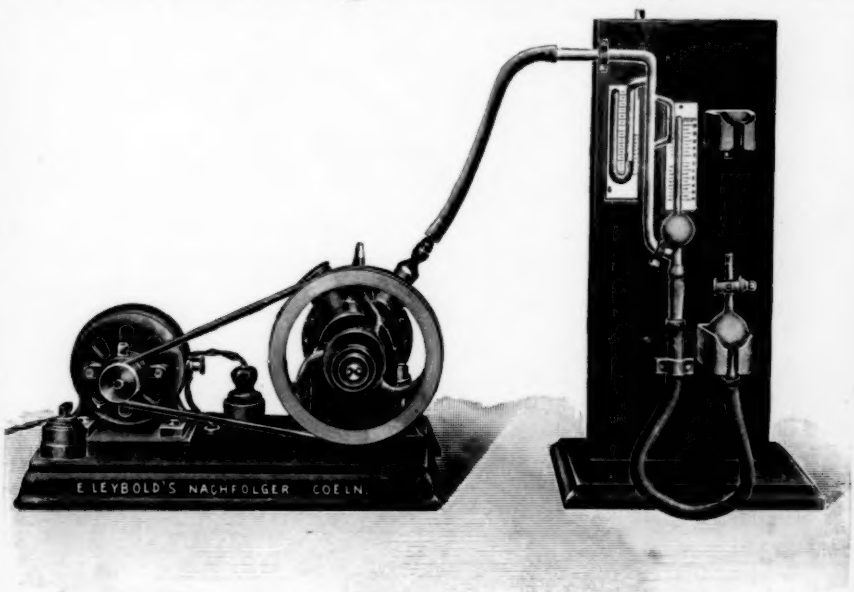

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chosen topics and questions for the pupils' use. These suggestions, map studies, and topics are very apt and afford the pupil an excellent recapitulation. The discussion of cotton in Chapter VI is a fair representation of the method employed in the treatment of the various industries. It is presented in clear language which brings the public in direct contact with the various phases of life in the cotton fields of the South and gains his interest and attention. The geographic basis of soil, climate, and location is shown in relation to the plantation and the products. The planting, picking, marketing, and shipping lead naturally to the manufacturing. The five pictures of this chapter are good types and clearly aid the interpretation of the text.

The contents of this book are in close accord with its title. Such a statement may appear contradictory but the meaning becomes obvious on the examination of some recent texts in this field in which old contents are issued under a new and popular title. The new movement in geography demands more of the knowledge of what people are doing and how they live. Many experimenters are trying to find suitable material which will be more typical of man's ways of living and what he does than the extensive physical geography or the unrelated place geography. This text is an excellent collection of material which will aid the elementary teacher in presenting geography along the lines of the greatest interest and value to the child. W. M. GREGORY.

Botany for High Schools, by George Francis Atkinson, Ph.B., Professor of Botany in Cornell University. Pp. xv + 493. Price, $1.25. Henry Holt and Company, New York. 1910.

The preface of this book states that it is "addressed to pupils in their first or second year in the high school" and that it is "assumed that botany may be the first science they study." The author may think he has written a book for pupils of these years in the high school, but anyone who has had experience in teaching these grades can tell him that he has failed to do what he intended. Professor Atkinson has written a good book, undoubtedly, one of the best for high school use now available, but it is really suitable for perhaps the fourth year of the high school or better still for freshmen in college.

Some things to be said in its favor are that it handles the subject in a straightforward and logical way, giving a large amount of information in a clear-cut way. The divisions by chapters and their arrangement are good. By far the best part of the book is "Part I, The Growth and Work of Plants." This part comes nearest to being within the powers of second year pupils—not first year. The chapter on economic plants is also good.

The general style of the book will prove dry and difficult reading for young pupils on account of the succinct and terse way the author has of stating the subject matter and also because he has freely used technical terms and words not in the vocabulary of young students in the high school. Although the author in the preface states that "relation to man" together with reproduction and nutrition are the "dominant aspects" of the book, the reviewer is much disappointed at the paucity of this very important phase of botany in the actual treatment of the subject in the book. The teaching of botany has been making great strides of late toward connecting botany with practical affairs. The author has left out much that would have been in keeping with the spirit of the times in botanical teaching and included very much that makes the subject difficult and distasteful to young pupils. If more than half of the 185 pages devoted to "General Morphology and Classification of Plants" had been omitted and chapters on plant propagation, plant selection and breeding, seed selection, gardening and other topics of kindred nature substituted and given adequate treatment, we should have had much to commend and little to condemn. W. W.

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Anharmonic Coördinates, by Lieutenant-Colonel Henry W. L. Hime. Pp. xiv+127. 1910. Price, $3.00. Longmans, Green & Co.

Fifty years ago Sir W. R. Hamilton, the inventor of Anharmonic Coördinates, gave a brief explanation of them in his *Elements of Quaternions.* Since that time no book has been written on the subject to the author's knowledge; and he believes that the method devised by a great British mathematician ought not to be altogether forgotten.

This book is confined to the application of the method to well-known geometrical theorems in the following chapters: Plane Geometric Nets; The Point; The Straight Line; Distances, Areas, and Angular Functions; The General Equation of the Second Degree; Special Conics; Tangential Equations; Cross Ratio; Transformation of Coördinates; The Circle; The Foci of a Conic; Miscellaneous Theorems. H. E. C.

An Elementary Course in Graphic Mathematics, by Matilda Auerbach, Instructor in Mathematics, Ethical Culture High School, New York City. Pp. 54. 1911. Price, 35 cents. Allyn & Bacon.

This little book gives an excellent presentation of the meaning and use of the graph. In the second chapter some practical uses of the graph are given under the following topics: (a) In surveying. (b) In keeping statistics, records, and as ready reckoners. (c) In representing formulas. (d) In the solution of problems involving the element of time. This is followed by a study of the function and the equation. The appendix contains forty-one tables of statistics for graphical representation.

"The object of this little book is threefold: first, to show the pupil some practical uses of the graphic method [*This is well done.—Math. Ed.*]; second, to plan a course in graphic algebra that will lead naturally and along pleasant paths to the work in the solution of equations [*Why must a course in graphic algebra lead to the work in the solution of equations? Are not the practical uses of more importance than the solution of simultaneous equations?—Math. Ed.*]; and finally, to save both teacher and pupil time and energy needed to hunt up suitable material." [*May the day soon come when teacher and pupil will realize the great value of the graph. Then there will be no desire for a saving of time and energy in finding suitable material. In fact, there is an abundance of such material within easy reach of teacher and pupil alike.—Math. Ed.*

The few pages of this book will give one a better working knowledge of the graph than some of the larger books on the subject that have appeared. H. E. C.

Plane and Spherical Trigonometry, by Arthur G. Hall, Professor of Mathematics, University of Michigan, and Fred G. Frink, Professor of Railway Engineering, University of Oregon. x+176 pages. 15x23 cm. Price, $1.40. Henry Holt & Co.

The amount and selection of material, method of presentation, and the large number of applications to physics and engineering, make this a very excellent text-book for a twelve weeks' course. The topics are presented in the following order: The acute angle, right triangles, logarithms, the obtuse angle, oblique triangles, the general angle, functions of two angles, analytic trigonometry, spherical trigonometry.

The slide rule is described, and it is suggested that it be used to check the work in the solution of problems. The carefully drawn diagrams, clear type, and opened spaced arrangement render the pages very attractive.

H. E. C.